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FHWA/IN/JHRP-83/12

DESIGN OF COMPACTED CLAY EMBANKMENTS FOR IMPROVED STABILITY AND SETTLEMENT PERFORMANCE

M. J. Goodman

J. L. Chameau

C. W. Lovell

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Interim Report

Design of Compacted Clay Embankments for Improved Stability and Settlement Performance

To:	H.L. Michael, Director Joint Highway Research Project	August 30, 1983	
		Project:	C-36-5M
From:	J.L. Chameau, Research Associate Joint Highway Research Project	File:	6-6-13

Attached is an Interim Report on the HPR Part II study titled "Improving Embankment Design and Performance". The report is entitled "Design of Compacted Clay Embankments for Improved Stability and Settlement Performance". It is authored by M.J. Goodman, J.L. Chameau and C.W. Lovell of our staff.

The report describes the application of the compacted clay investigation presented in previous interim reports to the design and analysis of compacted clay embankments. The alternatives of specifying compaction procedures or compaction results are compared, and a hybrid approach of compaction specification is introduced. Embankment slope design is illustrated for short and long term conditions. Computer programs to compute the magnitude and time-rate of settlement of compacted embankments are developed.

The report is submitted as partial fulfillment of the objectives of the study.

Respectfully submitted,

J.L. Chameau

J.L. Chameau
Research Associate

cc:	A.G. Altschaeffl	W. H. Goetz	C.F. Scholer
	J.M. Bell	G.K. Hallock	R.M. Shanteau
	W.F. Chen	J.F. McLaughlin	K.C. Sinha
	W.L. Dolch	R.D. Miles	C.A. Venable
	R.L. Eskew	P.L. Owens	L.E. Wood
	J.D. Fricker	B.K. Partridge	S.R. Yoder
	G.D. Gibson	G.T. Satterly	

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by

Martin J. Goodman
J. L. Chameau
and
C. W. Lovell

Joint Highway Research Project

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The contents of this report reflect the views of the authors who are responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views of policies of the Federal Highway Administration. This report does not constitute a standard, specification, or regulation.

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16. Abstract

This report is part of the embankment design and performance project conducted by the Joint Highway Research Project which was initiated to improve the ability of highway engineers to design embankments. It has two main objectives. First, it illustrates how the results of the compacted clay investigation can be used in the design and analysis of compacted clay embankments. Second, it completes the analysis package by supplying computer programs for the calculation of embankment settlement.

A hybrid method of specifying compaction which makes the in situ water content of the embankment soil equal to the optimum moisture content is introduced. This approach can help optimize the properties of compacted embankment soils. Embankment side slope design is illustrated for short and long term conditions using laboratory shear strength data. Geometric and probabilistic interpretation of the factor of safety are introduced as alternatives and/or supplements to the conventional strength factor of safety.

A methodology to predict the settlement of embankments is presented. Computer programs to compute the magnitude and time-rate of settlement are included. User's manuals for these programs are provided. Several improvements made to the program STABL are also documented.

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LIST OF SYMBOLS AND ABBREVIATIONS

a	–	lower bound of a probability density function
a	–	pore pressure parameter relating octahedral shear stress change to excess pore pressure change
a_v	–	coefficient of vertical compressibility of a soil
A_{ac}	–	Skempton pore pressure parameter relating excess pore pressure and deviatoric stress change in a triaxial test
A_{cr}	–	area of critical cross-section
A_f	–	Skempton pore pressure parameter at failure
A_1	–	term used by STABL to calculate the FS
A_2	–	term used by STABL to calculate the FS
A_3	–	term used by STABL to calculate the FS
A_4	–	term used by STABL to calculate the FS
A_5	–	term used by STABL to calculate the FS
b	–	upper bound of a probability density function
B	–	pore pressure parameter relating octahedral normal stress change to pore pressure change

B	-	number of blows in a compaction test
c	-	cohesion intercept
c_a	-	available cohesion
c_h	-	coefficient of consolidation in the horizontal direction
c_v	-	coefficient of consolidation in the vertical direction
C_c	-	compression index
CD	-	consolidated-drained triaxial test
CDF	-	cumulative distribution function
C_r	-	recompression index
CU	-	consolidated-undrained triaxial test
C_{v1}	-	coefficient of consolidation above a layer interface
C_{v2}	-	coefficient of consolidation below a layer interface
d	-	dimension used to compute the friction-circle factor of safety
dz	-	thickness increment
D	-	depth factor
e	-	void ratio
e	-	total compactive effort
e_o	-	initial void ratio
E	-	compactive effort
E(x)	-	expected value of x
f	-	coefficient of compaction
$\bar{f}$	-	average value of f

f_s	-	probability density function of the load
$f(x)$	-	probability density function of the variable x
F	-	towing force
F_R	-	cumulative distribution function of the resistance
FS	-	factor of safety
FS_c	-	factor of safety on the cohesion intercept
FS_{STABL}	-	factor of safety obtained with the Simplified Janbu FS
FS_β	-	factor of safety on a slope angle
FS_ϕ	-	factor of safety on the friction angle
$F_x(x)$	-	cumulative distribution function of the variable x
F_x^{-1}	-	inverse function of F_x
h	-	slice height
h_{eq}	-	height of horizontal earthquake force above bottom of a slice
H	-	embankment height
H	-	clay layer thickness
H_{cr}	-	value of slope height at which a slope reaches limit equilibrium
H_v	-	horsepower of a compactor
i	-	index number indicating position on the x axis
j	-	index number indicating position on the z axis
k	-	time step number

k	–	coefficient of compaction
k_v	–	vertical earthquake coefficient
k_h	–	horizontal earthquake coefficient
k_1	–	permeability above a layer interface
k_2	–	permeability below a layer interface
l_c	–	half-width of the cylindrical portion of a sliding mass
L	–	towing distance
M	–	weighting factor for calculating compactive effort
n	–	number of slices
n	–	number of strata
N_s	–	stability number
OCR	–	overconsolidation ratio
OCR_o	–	OCR at which excess pore pressure is zero
OMC	–	optimum moisture content
P_f	–	probability of failure
P	–	number of passes of a compactor
PDF	–	probability density function
P_s	–	compactive prestress
P1	–	perpendicular bisector of the first chord
P2	–	perpendicular bisector of the second chord
Q	–	randomly generated number between 0 and 1
r	–	correlation coefficient
r_o	–	dimension used to calculate stress change beneath an embankment

r_1 - dimension used to calculate stress change beneath an embankment

r_2 - dimension used to calculate stress change beneath an embankment

R - radius

R - strength

R_{min} - minimum value of strength

S - load

S - settlement

S - forward speed

S_{field} - the actual consolidation settlement of a compressible layer

S_{lab} - calculated value of consolidation settlement of a compressible layer assuming excess pore pressures in the field equal to the pressure developed in laboratory

S_{max} - maximum value of load

t - time

T - chord length of segments circumscribing a circle

T_L - time to travel the distance L

u - excess pore pressure

$u_{i,j,k}$ - excess pore pressure at the i^{th} x position, the j^{th} z position and the k^{th} time step

UU - unconsolidated-undrained triaxial test

U% - percent consolidation

$U(z)$ – the percent volume change due to saturation at the depth, z

v – variable used to compute the friction-circle factor of safety

ν – parameter used to define the beta distribution

V – mold volume

$V(x)$ – variance of x

ω – frequency of vibration of a vibratory compactor

$w\%$ – water content

W – compactor weight

x – the coordinate value on the x axis

χ – parameter used to define the beta distrbution

x_{c1} – x coordinate of the intersection of the first chord and its perpendicular bisector

x_{c2} – x coordinate of the intersection of the second chord and its perpendicular bisector

x_o – x coordinate of the center of the circle

x_1 – x coordinate of the first point on the first chord of a circle

x_2 – x coordinate of the second point on the first chord of a circle

x_3 – x coordinate of the second point on the second chord of a circle

y – the coordinate value on the y axis

$\bar{y}$ – vertical distance to bottom of a slice from the moment center

y_{c1} - y coordinate of the intersection of the first chord and its perpendicular bisector

y_{c2} - y coordinate of the intersection of the second chord and its perpendicular bisector

y_o - y coordinate of the center of the circle

y_1 - y coordinte of the first point on the first chord of a circle

y_2 - y coordinate of the second point on the first chord of a circle

y_3 - y coordinate of the second point on the second chord of a circle

Y - arbitrary function

Z - net work per vibratory cycle of a vibratory compactor

α - angle the bottom of a slice makes with the horizontal

α - parameter used to define the beta distribution

α_x - coefficient used in equation 4.19

α_z - coefficient used in equation 4.19

β - sideslope angle

β_{cr} - value of sideslope at which a slope reaches limit equilibrium

ΔC_a - cohesion force acting on the bottom of a slice

Δe_o - the difference between the initial void ratio in the soil sample and another soil stratum

ΔN - normal force acting on the bottom of a slice

ΔQ	-	surcharge load on a slice
ΔS_r	-	resisting force acting on the bottom of a slice
Δt	-	time increment
ΔW	-	slice weight
ΔU_α	-	water force acting beneath a slice
ΔU_β	-	water force acting on top of a slice
Δx	-	increment in the x direction
Δz	-	increment in the z direction
Δz_1	-	vertical grid spacing above a layer interface
Δz_2	-	vertical grid spacing beneath a layer interface
$\Delta \sigma$	-	stress change
$\Delta \sigma_v$	-	change in vertical stress
$\Delta \sigma_x$	-	change in normal stress acting in the x direction
$\Delta \sigma_z$	-	change in normal stress acting in the z direction
$\Delta \sigma_1$	-	change in major principal stress
$\Delta \sigma_3$	-	change in minor principal stress
$\Delta \tau_{xz}$	-	change in shear stress acting in the z direction on the plane perpendicular to the x axis

$\Delta\theta$	-	deflection angle of segments circumscribing a circle
γ	-	unit weight
γ_d	-	dry unit weight
$\gamma_{d\ max}$	-	maximum dry unit weight
γ_m	-	moist unit weight
γ_w	-	unit weight of water
Γ	-	gamma function
λ	-	nondimensionalized slope stability parameter
μ	-	consolidation settlement correction factor
μ_c	-	coefficient of variation of the cohesion intercept
ν	-	Poisson's ratio
ϕ	-	friction angle
ϕ_a	-	maximum available friction angle
ϕ_{req}	-	friction angle required for equilibrium
σ_{oct}	-	octahedral normal stress
σ'_p	-	effective preconsolidation pressure
σ_{sample}	-	the vertical pressure at the depth of the soil sample
σ'_v	-	effective vertical pressure
σ'_{vo}	-	the effective overburden pressure in a soil stratum
σ_y	-	yield strength
σ_1	-	major principal stress
σ_2	-	intermediate principal stress

σ_3 - minor principal stress

Σ - summation

τ_{oct} - octahedral shear stress

HIGHLIGHT SUMMARY

This study illustrates how the results of the compacted clay investigation can be best used in the design and stability analysis of compacted clay embankments. It also completes the analysis package by supplying computer programs for the calculation of embankment settlement.

The alternatives of specifying compaction procedures or compaction results are compared and an hybrid approach of specifying compaction that integrates the advantages of these two approaches is introduced. In the hybrid approach of compaction specification, the compactive effort is specified so that the corresponding optimum moisture content is equal to the expected compaction water content.

Embankment side slope designs are illustrated for short and long term conditions using laboratory compacted shear strength data. In these examples the embankment material is assumed to be compacted St. Croix clay and the strength parameters for short and long term conditions are obtained from the reports by Weitzel and Lovell (1979) and Johnson and Lovell (1979), respectively. Several improvements made

to the STABL program during the course of this study are presented:

- The Simplified Bishop factor of safety option was recoded to correct various difficulties discovered by STABL users.

- Recommendations are made to avoid common errors in the use of STABL (surfaces with unacceptable shapes, direction limits on benched slopes, direction of surface generation, etc.).

- A methodology was developed to adjust the Simplified Janbu factor of safety (used in several STABL options) to more familiar definitions of the factor of safety.

Geometric and probabilistic interpretation of the factor of safety are introduced to demonstrate their usefulness as alternatives and/or supplements to the conventional strength factor of safety. The probabilistic approach takes into account the variability of material parameters and provide the reliability of the slope corresponding to the computed factor of safety.

Computer programs are developed to compute the magnitude of settlement that occur within the embankment itself as well as of the consolidation settlement of fine grained soil layers below the embankment. Programs are also provided to estimate the time-rate of consolidation settlement

of the compressible soil layers beneath an embankment. These programs, in conjunction with STABL, form an analysis package for the design of embankments. User's manuals and listings are given for all these computer programs.

I - INTRODUCTION

This report is a part of Purdue's embankment performance project which was initiated to improve the ability of highway engineers to design highway embankments.

This project is composed of two separate objectives. One objective is the investigation of the strength and compressibility parameters of compacted St. Croix clay; a highly plastic residual clay of Southern Indiana. The second objective of this project entails the development of an analysis package to predict the settlement and stability performance of embankments.

Compacted Clay Investigation

The study of the parameters defining the behavior of compacted St. Croix clay was performed in two phases. The first phase consisted of tests performed on clay samples that were prepared to simulate standard laboratory compaction specifications. This testing program included the following separate investigations:

1) Compressibility and prestress behavior of St. Croix clay (DiBernardo, 1979).

2) Unconsolidated-undrained (UU) shear strength behavior of St Croix clay (Weitzel, 1979).

3) Consolidated-undrained (CU) shear strength of St. Croix clay (Johnson, 1979).

Parameters defining the behavior of field compacted clay are different than those obtained with laboratory testing because of the difference in compactive effort between field compactors and laboratory compaction tests. Therefore, in the second phase, the tests of the first phase were repeated on samples compacted in the field with two different rollers (Lin, 1981 and Liang, 1982). The swell pressure of both laboratory and field compacted clay was studied by Terdich (1981). These studies provide a unique look at the correlation between the behavior of field and laboratory compacted clays.

Analysis Package

Purdue University has a long standing interest in the development of user-oriented slope stability computer programs. One of the first developments was the SLOPE program package (Carter, 1971) consisting of four separate programs. The subsequent development was the STABL program (Siegel, 1975). This program can evaluate the factor of safety of slopes of almost any description and shape. Boundary surcharge loads and pseudo-static earthquake forces may also be

included. The most significant feature of STABL is its ability to automatically create randomly generated surfaces to help the user search for the minimum value of the factor of safety.

STABL was further developed by Boutrup (1977). Her improvements included:

1) The addition of specialized search routines for simulation of block shaped failure surfaces with active and passive wedges.
2) The inclusion of the Simplified Bishop factor of safety.
3) Changes in the input of pore water pressure that allow the simulation of artesian pressure.

STABL is used on a regular basis by the Indiana Department of Highways for routine evaluation of slope stability.

The most recent of Purdue contributions to the field of slope stability is the development of a factor of safety that takes into account the three-dimensional nature of limit equilibrium surfaces (Chen, 1981). Currently, this method is programmed only for simple slope shapes and failure surfaces.

Report Organization

The purpose of this report is twofold. First, it completes the analysis package by supplying computer programs for the calculation of embankment settlement. Second, it illustrates how the results of the compacted clay investigation may be best used in the design of compacted clay embankments.

Chapter II provides an overview of compaction specification. The alternatives of specifying compaction procedures or compaction results are compared. A hybrid approach of specifying compaction that integrates the advantages of these two approaches is introduced.

Chapter III covers a variety of topics pertinent to the subject of slope stability including:

1) Use of the STABL program for slope stability analysis. The discussion includes recent corrections made to the Simplified Bishop option.
2) Strength parameters of compacted clays to be used when assessing the factor of safety and stability of compacted clay embankments.
3) Discussion of geometric and probabilistic interpretations of the factor of safety.

Chapter IV deals with the settlements caused by the construction of an embankment. User-oriented computer programs to facilitate the computation of the magnitude and time-rate of consolidation settlement are included.

Chapter V presents conclusions of the work that was done and suggestions for further research.

II - COMPACTION SPECIFICATION

Compaction is the densification of soil by the application of mechanical energy. Densification is achieved by reduction of the size and number of air voids in the soil. As the volume of the voids is reduced, the shear strength of the soil is increased and its permeability is decreased. The increase in shear strength and the reduction in permeability are two factors that make compaction a good technique for constructing highway embankments and dams.

Specifying an adequate level of compaction and range of water content indirectly assures that the soil will have a relatively high shear strength and a low compressibility. This is fortunate because these soil properties are difficult and time consuming to measure on a routine basis.

It is important to be able to quantify the level of compaction to determine if the compaction is adequate. In his pioneering work, Proctor showed that the level of compaction depends on the compactive effort, E, i.e., the amount of mechanical energy imparted into a unit volume of

soil (Proctor, 1933). For a given compactive effort, the dry density , γ_d, that can be achieved varies with the water content of the soil (Figure 2.1). There is a value of the water content called the optimum moisture content , OMC, at which the dry density has a maximum value. The soil shear strength is maximized and the soil permeability in service is minimized at or near this water content. The OMC is a function of the compactive effort that varies along a "line of optimums" (Figure 2.2). In general, the dry density will increase and the OMC will decrease as the compactive effort increases.

The concept of compactive effort provides the basis for evaluating the level of compaction. At a specified value of compactive effort, there is a maximum achievable dry density , $\gamma_{d\ max}$, corresponding to the OMC which can be achieved. At water contents other than the OMC, γ_d that is achieved will be less than $\gamma_{d\ max}$. Therefore, it is convenient to define the "percent compaction" as the ratio of γ_d to $\gamma_{d\ max}$.

Specification of Procedure

The basic philosophy of specifying the compactive procedure is that if a certain procedure is followed, the compaction is assumed to be satisfactory. Typically, this is achieved by construction of a test pad. Each time the compactor passes over a soil lift in the test pad, the soil

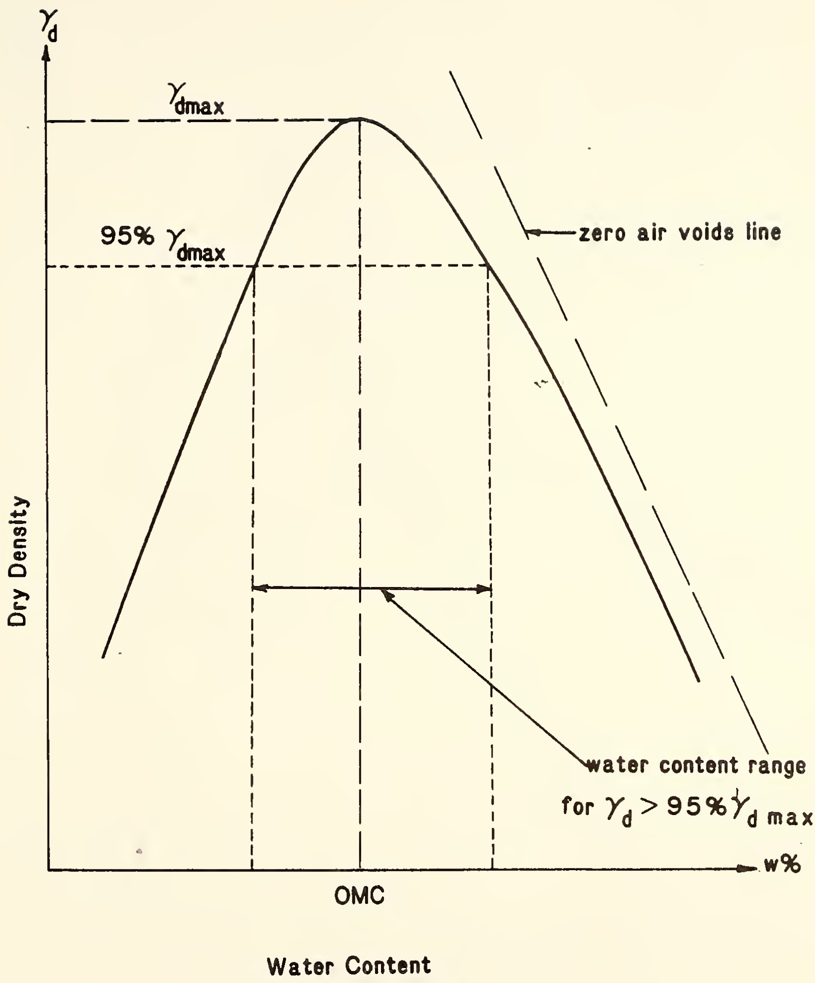


FIGURE 2.1 MOISTURE–DENSITY RELATIONSHIP OF A COMPACTED CLAY

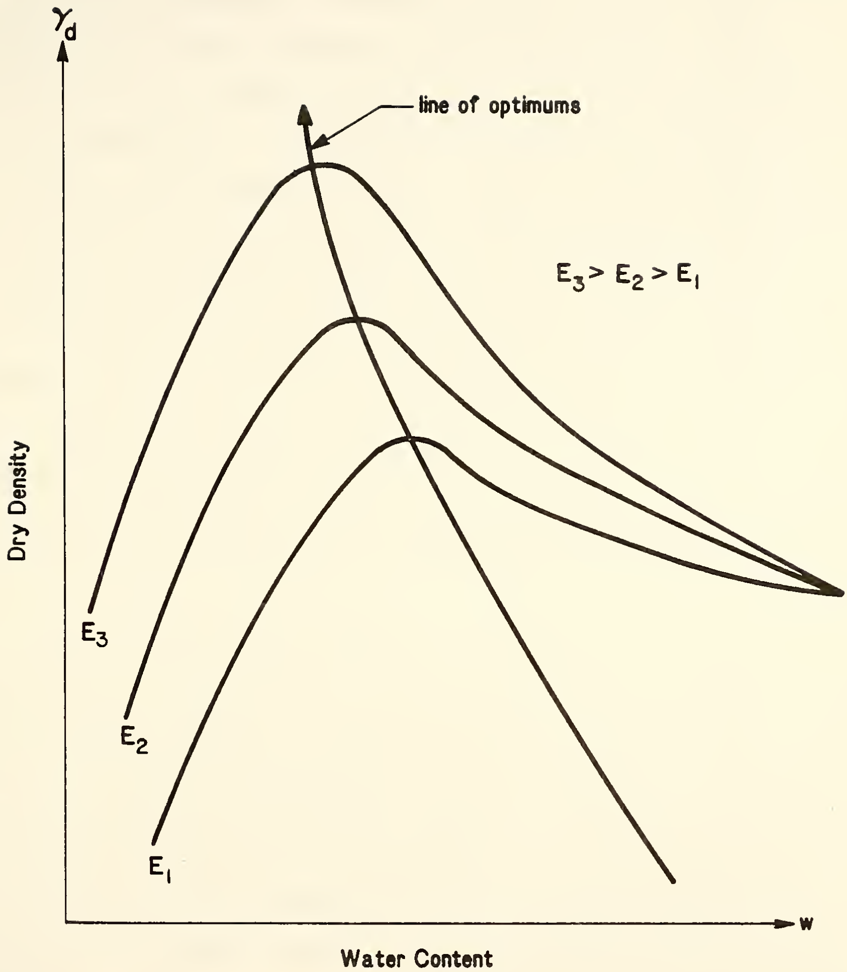


FIGURE 2.2

VARIATION OF THE MOISTURE – DENSITY RELATIONSHIP OF A COMPACTED CLAY VS. COMPACTIVE EFFORT

density is recorded using either a sand cone test or a nuclear density device. The measured density is then plotted versus the number of passes of the compactor. The result is a "density growth curve" (Figure 2.3). The density growth curve indicates the effectiveness of each pass of the compactor. The increase in density diminishes with each pass of the compactor until a pass generates a negligible density increase. Beyond this point, higher densities can only be obtained by using machines that impart more compactive effort. Therefore, although the number of passes should be specified to provide a large fraction (perhaps more than 95%) of the achievable density, the specified number of passes should be less than the value at which each subsequent pass creates only a negligible increase in density.

It is recommended that the density growth curve be developed for various lift thicknesses, various rates of advance of the compactor, and different compactors. This makes it possible to determine the best compactor and the optimum mode of operation for a given job. The performance study done at Purdue (Terdich, 1981) is a good example of the use of the density growth curve. This study showed that a Caterpillar Model 825 tamping compactor is significantly more effective than a RayGo Rascal Model 420C Vibratory compactor to compact St. Croix clay. Advantages of specifying the procedure include:

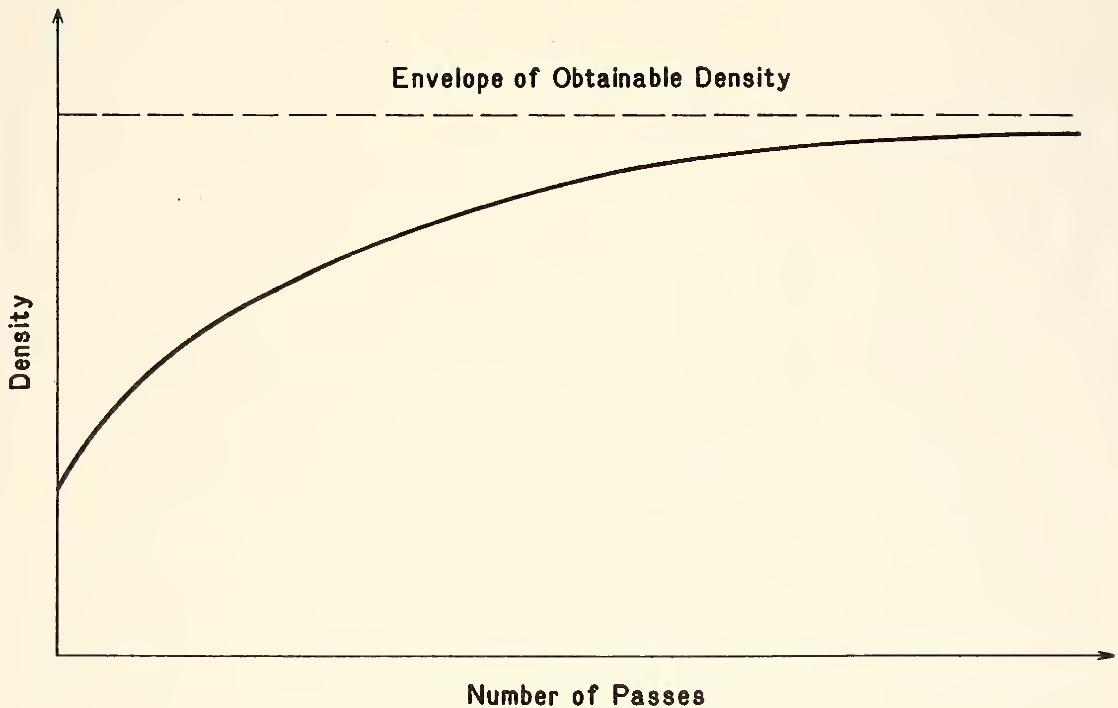


FIGURE 2.3 DENSITY GROWTH CURVE

1) Only limited testing is required to provide quality control of the compaction.

2) The contractor has the assurance that if he operates in accordance with the specified procedure, he will obtain the necessary compaction. This helps reduce the adversarial aspect of the relationship between the contractor and the engineer.

Specification of Results

On jobs where the compacted soil is expected to be quite variable, specifying the procedure may not be practical because the level of compaction will change in an unknown fashion from soil to soil, even if identical procedures are used. When this occurs, the results of the compactive operation must be specified.

Specification of results entails two stages. The first stage involves the development of a moisture-density relationship from compaction tests run on soil samples compacted in the laboratory. The laboratory compaction should produce similar moisture-density results as those produced by the compactive effort that is expected in the field. The relationship between the moisture and the density is called the "control curve". The second stage involves the comparison of field measurements of the dry density with the maximum dry density on the control curve. Generally, if the percent compaction is between 95% and 100%, the compaction is judged

to be satisfactory. If the percent compaction is consistently greater than 100%, the laboratory test is probably not imparting as much compactive effort as the compactor. Conversely, if the percent compaction is consistently less than 95%, either the lab compaction is imparting more compactive effort than the field compactor is able to deliver, or more passes of the field compactor are required.

To adjust the laboratory compaction technique to approximate the effects of the compactive effort in the field, it is helpful to quantify the value of the compactive effort of a desired compactor (Selig, 1971). Selig proposed that all compactors be represented by a single drum towed by a separate machine. In this representation, the total compactive effort of the roller for smooth wheel, pneumatic tire, and tamping compactors is

$$e = F \cdot L \cdot P \tag{2.1}$$

where

e = total compactive effort (ft·lb)

F = towing force (lb)

L = distance towed (ft)

P = the number of passes over the distance L

This implies that all of the compactive effort of these compactors is provided by the towing unit through the drawbar.

In contrast, the compactive effort of vibratory rollers is imparted by the vibratory energy of the roller drum. The effect of the towing unit on the drawbar is assumed to be negligible. Therefore, the total compactive effort of a vibratory roller is

$$e = Z \cdot \omega \cdot T_L \tag{2.2}$$

where

Z = net work per cycle (ft·lb/cycle)

ω = frequency of vibration (cycles/min)

T_L = time to travel the distance L (min)

Although equations 2.1 and 2.2 both assume that all of the mechanical energy of the roller is transformed into compactive effort, only a fraction of the mechanical energy is actually transformed. The ratio of the compactive effort to the mechanical energy is called the "efficiency". The compactive effort per unit volume may be expressed with quasi-analytic expressions which are specific to a compactor. These expressions are given in Table 2.1. All these expressions contain a coefficient of rolling friction, f. Like compactive effort, this coefficient has almost never been determined experimentally (Selig, 1971). To overcome this, Selig estimated the compactive effort of several types of compactors by comparing the densities that they could achieve with the densities obtained in Standard and Modified Proctor compaction tests. Substituting these values into the expressions in Table 2.1, he evaluated the values of f.

Table 2.1 Expressions for the Compactive Effort of Various Types of Compactors (after Selig, 1971)

Roller Type	Compactive Effort
Smooth Wheel	$E = \frac{fWP}{Bt}$
Pneumatic	$E = \frac{fWP}{ht}$
	or
	$E = \frac{fWP}{Bt}$ if $d < 2b$
Tamping	$E = \frac{fW\pi(d + 2)}{k_o + cNA}$
	or
	$E = \frac{fWP}{Bt}$
Vibratory	$E = \frac{375H_vP}{SBt}$
	or
	$E = \frac{fWP}{Bt}$
	where $f \sim \frac{375H_v}{WS}$

Table 2.1 (continued)

A = contact area of tamping foot (ft^2)
B = roller width (ft)
b = tire width (ft)
c = foot area correction factor > 1.0
D = diameter of roller drum (ft)
E = compaction effort per unit volume ($ft \cdot lb/ft^3$)
f = coefficient of compaction
H_v = horsepower of vibrator engine
h = nb for $d \geq 2b$
= $b + (n-1)d$ for $d < 2b$
k_o = overlap correction factor ≤ 1.0
l = tamping foot length (ft)
N = number of tamping feet
n = number of tires
P = number of passes
S = forward speed (miles/hour)
t = compacted lift thickness (ft)
W = total weight of compactor (lb)

Therefore, Selig's f coefficient is not actually a coefficient of rolling resistance, but a general purpose correction factor called the coefficient of compaction, which reflects the number of compactor passes, the soil lift thickness, and the soil type. The range of values of f and the recommended average design values are given in Table 2.2.

The relations for compactive effort in Table 2.1 assume that the compactive effort delivered to the soil is linearly proportional to the number of passes of the compactor. In practice, this is not the case. As the number of passes of the compactor increases, the densification per pass diminishes. This implies that as the number of passes increases, a lesser amount of the work done by the compactor is transformed into compactive effort. Selig (1971) showed that this phenomenon could be incorporated in his model with the following expression for the f parameter:

$$f_i = kt/P \tag{2.3}$$

where

f_i = coefficient of compaction after the i^{th} pass

k = compaction constant

t = compacted layers thickness

P = number of passes

It follows that the average value of f over P passes is

Table 2.2 Values of f and k for use in Table 2.1 and Equation. 2.6 (after Selig, 1971)

Roller Type	Soil Type	f	f_{avg}	k
Sheepsfoot	2	0.20–0.50	0.35	1.9
	3	0.05–0.15	0.10	1.6
Pneumatic	1	0.05–0.25	0.15	1.1
	2	0.10–0.30	0.25	1.3
	3	0.05–0.25	0.15	1.1
	4	0.05–0.30	0.15	1.4
Smooth Wheel	1	0.20–0.50	0.35	1.3
	2	0.15–0.25	0.15	1.1
	3	0.20–0.50	0.25	1.1
	4	0.10–0.40	0.30	1.3
Vibratory	1	0.20–0.40	0.30	2.7
	2	0.15–0.40	0.25	2.2
	3	0.30–0.60	0.40	2.5
	4	0.50–1.00	0.80	2.7
Segmented Pad	2	0.10–0.30	0.20	0.9
	3	0.10–0.25	0.15	1.1

Soil Type	Description
1	sand
2	silty clay
3	gravel-sand-clay
4	crushed stone

$$\bar{f} = \frac{1}{P}\sum_{i=1}^{P}\left(\frac{kt}{i}\right) \tag{2.4}$$

or

$$\bar{f} = k \cdot t \cdot M/P \tag{2.5}$$

where

$$M = \sum_{i=1}^{P}\frac{1}{i}$$

Substituting $\bar{f}$ into the basic expression for compactive effort (Table 2.1) yields:

$$E = k \cdot W \cdot \frac{M}{B} \tag{2.6}$$

Suggested values of k for use in equation 2.6 are given in Table 2.2. Unlike the equations in Table 2.1, equation 2.6 does not reflect the effect of variables specific to individual compactors such as tire spacing, tamping foot length, vibrator horsepower and operation speed. This simplification was necessary due to insufficient data. Even so, the compactive effort should be estimated with equation 2.6 because, unlike the equations in Table 2.2, it simulates the dimunition in compaction per pass. The two formulations of the compactive effort equations are compared in the following two examples for the compactors which were used in the Purdue embankment performance study (Terdich, 1981).

<u>Example 2.1</u>

It is desired to use a Caterpiller Model 825 segmented

pad tamping roller to compact one foot lifts of St. Croix clay. The specifications for this roller are provided in Table 2.3. Determine the variation of compactive effort delivered to the soil with the number of passes of the roller.

The total weight of the roller is approximately 60,000 pounds. The compaction is performed with four drums each with a 44.5 inch width. Using the expression for compactive effort of tamping rollers given in Table 2.1, the average compactive effort is:

$$E = \frac{(f)(60,000)(P)}{(44.5/12)(1)} = 16,180\ fP$$

The coefficient f may be assumed to be approximately 0.2 (using the value for silty clay in Table 2.2). It follows that:

$$E = 3236\ P \quad (ft \cdot lb/ft^3)$$

Alternately, the compactive effort may be evaluated using equation 2.6. The k coefficient may be approximated by the value of k for a segmented pad compactor operating on silty clay, i.e., k = 0.9. Substituting into equation 2.6 yields

$$E = \frac{(0.9)(60,000)(M)}{(44.5/12)} = 14562\ M$$

The results of this calculation are given in Table 2.4. The results of the two expressions for compactive effort are

Table 2.3 Specifications of Compactors Used in Purdue's Embankment Performance Project (after Terdich, 1981)

Caterpillar Model 825

Dimensions	
Length, with dozer	23 ft, 4 in.
Width, w/o clearers	11 ft, 11 in.
w/o dozer	12 ft, 6 in.
w/o dozer	13 ft, 7 1/2 in.
Wheelbase	140 in.
Weight (shipping)	
w/o dozer	59,000 lb.
with dozer	63,000 lb.
Number of Drums	4
Number of Pads/Drums	65
Each drum width	44 1/2 in.
Max. ballast	244 U.S. Gal
Bulldozer Dimensions	
Length	14 ft.
Height	40 1/2 in.
Maximum Speeds	Gear MPH
	1 3.1
	2 7.0
	3 17.0

Table 2.3 (continued)

RayGo Rascal Model 420C

Dimensions	
Length, with blade	18 ft, 9 in.
Width	9 ft
Wheelbase	9 ft
Weight	25,160 lb.
Vibration Drive	Hydraulic, Direct Drive
Frequency	1100 to 1500 rpm
Dynamic Force	32,000 lb
No. of Drums	1
No. of Pads/Drums	140
Each drum width	84 in.
Maximum Speeds	Gear / MPH
	1 / 4
	2 / 6
	3 / 8

Table 2.4 Calculations - Examples 2.1 and 2.2

Passes	M	Compactive Effort ($ft \cdot lb/ft^3$) E = 14562 M	E = 7907 M
2	1.50	21843	11861
4	2.08	30289	16447
2	1.50	21843	11861
4	2.08	30289	16447
6	2.45	35676	19373
8	2.72	39608	21508
10	2.93	42666	23169
12	3.10	45142	24513
14	3.25	47326	25699
16	3.38	49219	26727

compared in Figure 2.4. The equation for compactive effort of Table 2.2 underestimates the compactive effort up to 15 passes because it does not simulate the dimunition in compaction per pass. Note, that the values of compactive effort for this roller are generally intermediate to the total values of the Standard and Modified Proctor compaction tests.

Example 2.2

Repeat Example 2.1 for a RayGo Rascal Model 420C vibratory roller. Specifications are provided in Table 2.3. The width of the drums is 84 inches. The total weight of the compactor is 25,160 pounds. Assume that the roller proceeds at 1.5 miles per hour. The f coefficient is estimated to have the value given in Table 2.2 for a vibratory roller on silty clay, i.e., $f = 0.25$. The horsepower of a vibrator is (Table 2.1):

$$H_v = 0.0027\ fWS \tag{2.7}$$

Therefore, the horsepower of this particular vibrator is:

$$H_v = (.0027)(.25)(21{,}600)(1.5) = 21.87\ \text{hp}$$

The expression for the compactive effort of a vibratory roller from Table 2.1 is:

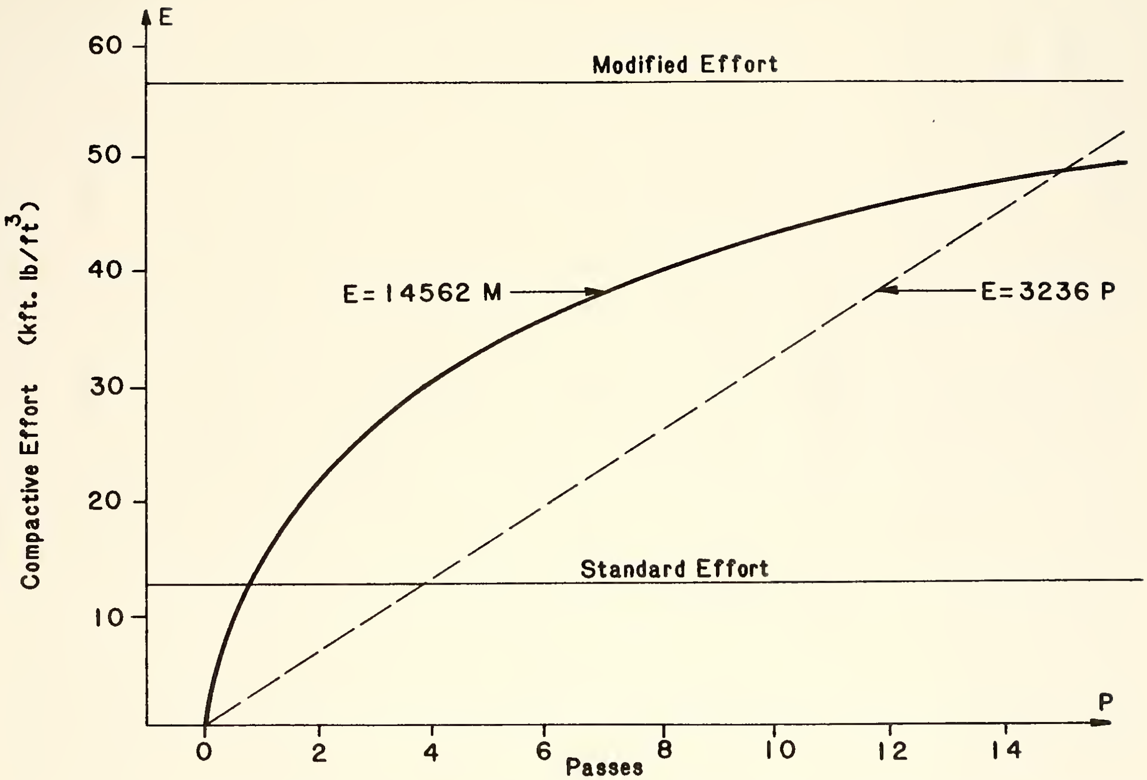


FIGURE 2.4 ESTIMATED COMPACTIVE EFFORT OF A CATERPILLER 825 COMPACTOR

$$E = \frac{(375)(21.87)(P)}{(1.5)(84/12)(1)} = 781\ P$$

Alternately, the compactive effort may be obtained from equation 2.6. Note that equation 2.6 employs the static weight instead of the dynamic force. This discrepancy is accounted for in the k coefficient (k = 2.2, see Table 2.2). Substituting into equation 2.6 yields:

$$E = \frac{(2.2)(25,160)(M)}{(84/12)} = 7907\ M$$

The results of this calculation are presented in Table 2.4 and Figure 2.5. Although equation 2.6 does not include speed of the roller, it does account for the reduction in compaction per pass as compaction proceeds. Therefore, as was the case with the tamping roller, the compactive effort of the vibratory roller is better predicted by equation 2.6 than the expression in Table 2.1. This particular compactor will yield the value of total Standard compactive effort after approximately two passes (See Figure 2.5). A comparison of Figures 2.4 and 2.5 indicates that the Caterpiller tamper is expected to be more effective than the Rascal Vibratory roller for compacting clay.

Assuming a selection of compactor and its use, based upon experience, a laboratory test may be selected, assuming that the total energy in the field and laboratory are equal. The equation for calculating the compactive effort in a compaction test is:

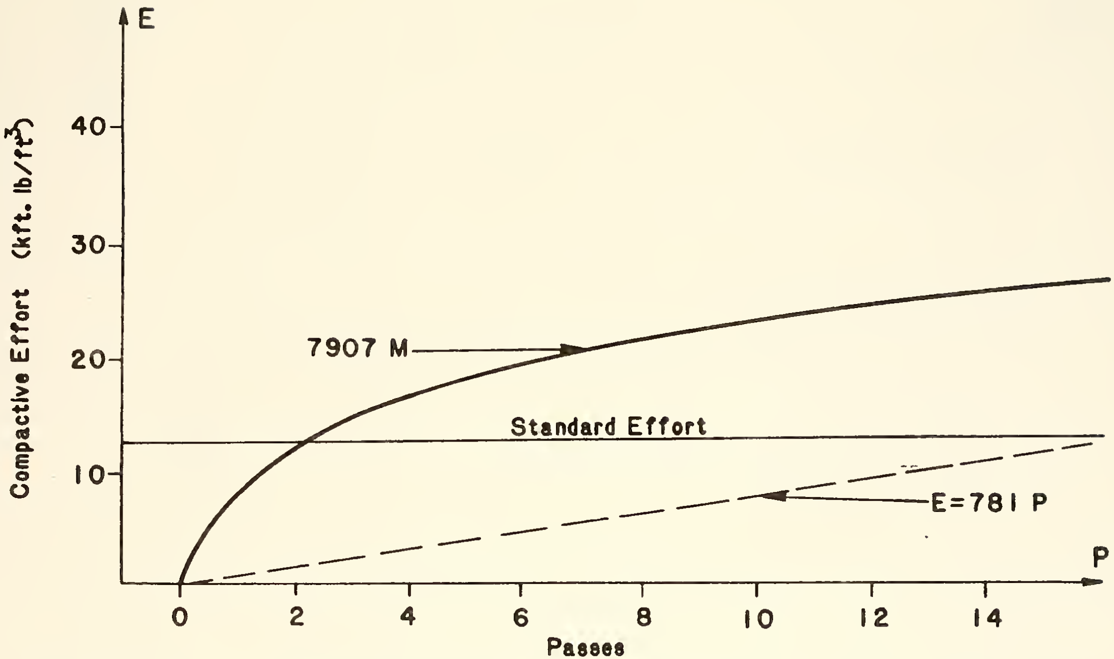


FIGURE 2.5 ESTIMATED COMPACTIVE EFFORT OF A RAYGO MODEL 420 C COMPACTOR

$$E = H \cdot W \cdot B \cdot L/V \tag{2.8}$$

where

E = compactive effort

H = height of fall of the hammer

W = weight of the hammer

B = number of blows per soil layer

L = number of soil layers

V = volume of the mold

If equipment is already available to perform an impact compaction test, the volume of the mold, the weight of the hammer, and the height of fall are predetermined. If the number of layers is set as in the Standard and Modified compaction tests, the only unspecified variable is the number of blows per layer. This may be adjusted as desired to approximate the rollers compactive effort. Rearranging equation 2.8, the required number of blows is:

$$B = \frac{V \cdot E}{H \cdot W \cdot L} \tag{2.8a}$$

<u>Example 2.3</u>

It is desired to prepare a soil sample to the level of

compaction expected from the Rascal compactor after four passes (Example 2.2). The sample will be compacted with the equipment used for performing standard compaction tests:

$V = 1/30\ ft^3$

$L = 3$ layers

$W = 5.5$ lb.

$H = 1.0$ ft.

From Table 2.4, the energy corresponding to four passes is $16447\ ft \cdot lb/ft^3$. Therefore the number of blows/layer should be

$$B = \frac{(1/30)(16447)}{(1)(5.5)(3)} \sim 33 \text{ blows/layer}$$

This represents approximately 25% more compactive effort than standard compaction.

The Hybrid Approach

Figure 2.2 shows that the OMC varies along the line of optimums, and as the compactive effort increases, the OMC decreases. When a compactor imparts a compactive effort associated with an OMC equal to the water content in the field, the compaction process is efficient. Therefore, the specified compactive effort should correspond to an OMC equal to the expected compaction water content. This combination of specification of results and specification of procedure involving a field compactive effort is called the hybrid approach.

<u>Example 2.4</u>

The insitu water content of St. Croix clay that is to be used to construct an embankment is 20%. The variation of the compactive effort vs. the OMC for laboratory compaction is shown in Figure 2.6. The compaction water content is equal to the OMC for a compactive effort of approximately 25,000 ft·lb/ft^3. Assuming that the efficiency of the compaction in the field is approximately equivalent to laboratory compaction, three passes of Caterpillar Model 825 should be sufficient to impart this compactive effort to the soil (Figure 2.4). Once the choice of compactor has been made, a density growth curve should be developed on a test section to check the prediction.

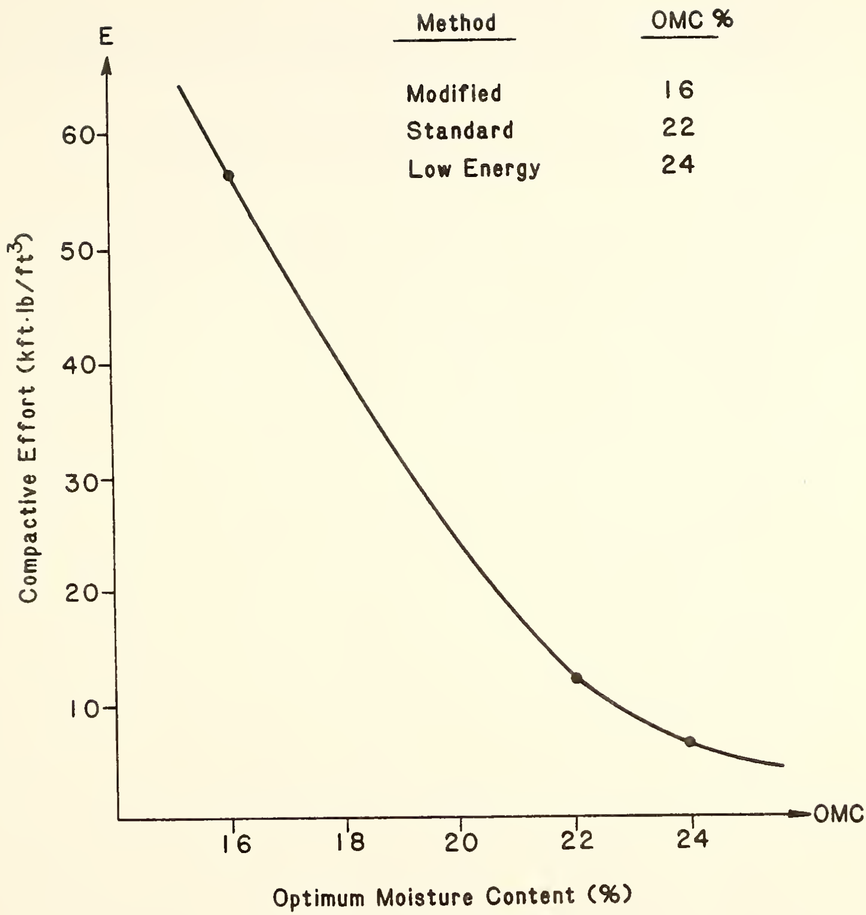


FIGURE 2.6 VARIATION OF OPTIMUM MOISTURE CONTENT VS. COMPACTIVE EFFORT FOR ST. CROIX CLAY (AFTER DI BERNARDO, 1979)

III - SLOPE STABILITY CONSIDERATIONS FOR EMBANKMENT DESIGN

The Concept of the Factor of Safety

The first criterion to be satisfied in the design of a compacted embankment is the stability of the embankment side slopes. Typically, the relative stability of a slope is assessed with a factor of safety obtained by limit equilibrium analysis. The factor of safety is defined as the ratio of available strength to applied shear stress along a surface of unit thickness beneath the free surface of the slope. Each slope has a family of such surfaces. The surface with the minimum factor of safety is referred to as the critical surface. If the factor of safety on the critical surface is greater than one, the slope is considered stable. Conversely, if the factor of safety on the critical surface is less than one, the slope is considered unstable. Since the value of the factor of safety that controls a design is the minimum value, the minimum factor of safety will be referred to as the factor of safety throughout this chapter, unless specified otherwise.

Factors that complicate the relationship between the

critical surface and the expected failure surface include:

(1) The dependence of the position of the critical surface on the factor of safety.

(2) The deviations of the soil shear strength behavior from the mathematical models used to quantify it.

(3) Errors inherent in the way slope stability analysis methods calculate the normal stresses along the trial surface.

(4) Additional resistance due to end effects in actual three-dimensional surfaces.

In structural mechanics, the factor of safety is computed by comparing the resistance of a structure to a load that is applied to the critical surface. For example, if an axial load is applied to the prismatic elasto-plastic bar shown in Figure 3.1, the critical surface will be the cross-section with the smallest area. Therefore, the factor of safety will be

$$FS = \sigma_y \cdot A_{cr}/P \tag{3.1}$$

where

σ_y = yield strength of the bar

A_{cr} = area of the smallest cross-section of the bar

P = the applied load

The factor of safety in this case is a measure of the proximity to failure of the cross-section on which the bar is

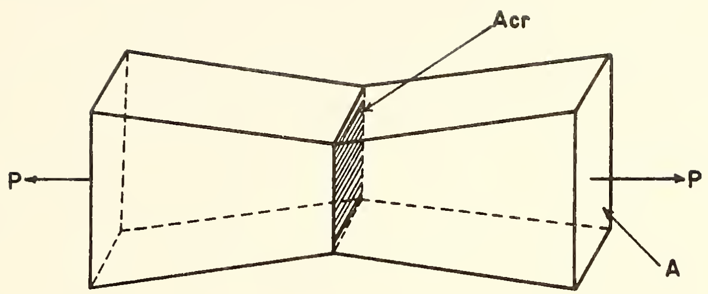


FIGURE 3.1 CRITICAL SURFACE OF AN AXIALLY LOADED TAPERED PRISMATIC BAR

expected to fail if the load is increased or the yield strength lowered. This is not the case for the factor of safety used in slope stability. Consider the embankment shown in Figure 3.2a, whose geometry is defined by the slope height, H, and the slope angle, β. The shear strength is assumed to be a constant everywhere in and under the embankment. If the value of the soil density increases, the factor of safety will decrease and the critical surface will move progressively deeper under the embankment provided that the soil strength does not increase because of the density increase (Figure 3.2b). Each value of the factor of safety corresponds to a different critical surface. This implies that the slope is expected to fail along a different surface than the critical surface.

Typically, a shear strength envelope is obtained by using peak values of the deviatoric stress from a triaxial test run to simulate insitu conditions. However, the soil in the slope may only be able to sustain a reduced deviatoric stress because of strain softening. Since the states of stress and strain vary greatly from position to position within an embankment, it is unlikely that the maximum values of the strength envelope can be developed simultaneously along any trial surface on which the factor of safety is to be evaluated. Therefore, using a strength envelope based on peak values of the deviatoric stress will usually result in an overestimate of the factor of safety. Although the

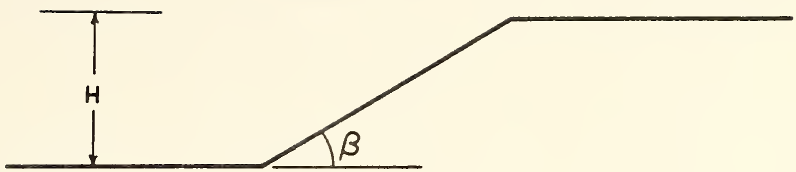


FIGURE 3.2a) SIMPLE SLOPE

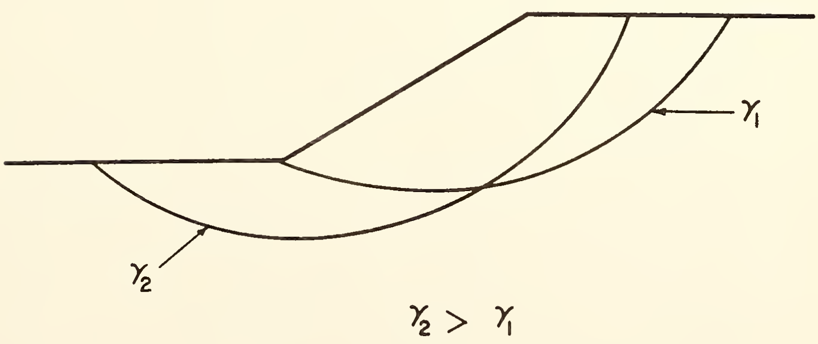
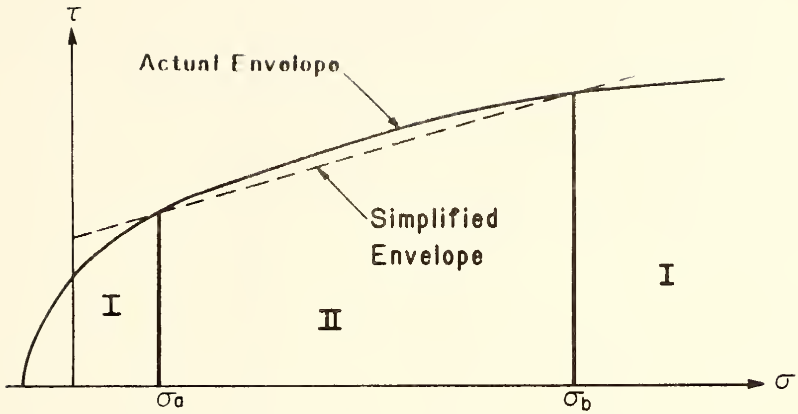


FIGURE 3.2b) CRITICAL SURFACES ON A SIMPLE SLOPE

unconsolidated-undrained (UU) shear strength envelope of unsaturated compacted clay is nonlinear if the water content is less than the OMC (Weitzel, 1979), it may be represented by a straight-line Mohr-Coulomb envelope (Figure 3.3). This simplification underestimates the strength along portions of the surface where the normal stress (σ) is in the $\sigma_a \leq \sigma \leq \sigma_b$ range (Zone II) and overestimate the strength outside these limits (Zone I). The net effect can cause the factor of safety to be overestimated or underestimated depending on the percentage of the trial surface that is in Zone I or Zone II.

Generally, the parameters that define the density and strength of soil in and under a slope will vary with position. In order to analyze such cases, it is current practice in slope stability analysis to resort to a method of slices. Many such methods exist, each with its own set of assumptions to overcome the indeterminacy of the problem. These methods assume that the local factor of safety, i.e., the factor of safety on a particular slice, is equal to the global factor of safety of the soil mass above the trial surface. This is only exactly true at limit equilibrium. Consider the unsaturated slice in Figure 3.4. Assuming that the slices are of a negligible width, the forces on the sides of the slice are equal and opposite. The summation of forces in the direction normal to the bottom of the slice gives:



Zone I, III Mohr Coulumb envelope overestimates shear strength
Zone II Mohr Coulumb envelope underestimates shear strength

FIGURE 3.3 COMPARISON OF AN ACTUAL ENVELOPE AND A MOHR-COULUMB REPRESENTATION

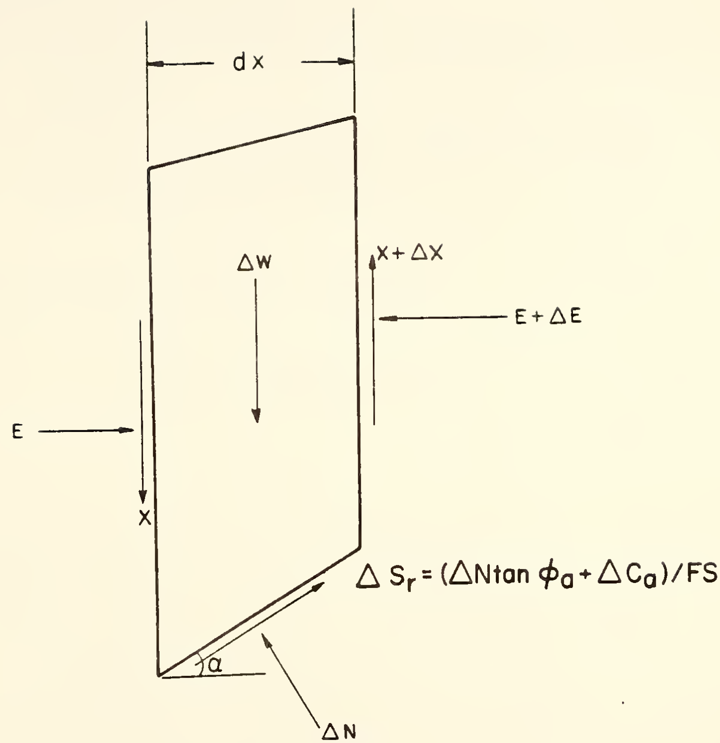


FIGURE 3.4

FORCES ON A SLICE (AFTER SIEGEL, 1975)

$$\Delta N = [\Delta W - \Delta S_r \cdot \sin\alpha]\cos\alpha \tag{3.2}$$

where

ΔN = the force normal to the bottom of the slice

ΔW = the weight of the slice

ΔS_r = the resisting force along the bottom of the slice

α = the angle that the bottom of the slice makes with the horizontal axis.

If a Mohr-Coulomb envelope is used, then

$$\Delta S_r = [\Delta N \cdot \tan\phi_a + \Delta C_a]/FS \tag{3.3}$$

where

$\tan\phi_a$ = slope of the Mohr-Coulomb envelope

ΔC_a = the product of the arc length of the bottom of the slice and the Mohr-Coulomb cohesion intercept

FS = local factor of safety of the slice.

Substituting equation 3.3 into equation 3.2 and rearranging yields:

$$\Delta N = \frac{\Delta W \cdot \cos\alpha - \Delta C_a \cdot \sin\alpha \cdot \cos\alpha/FS}{1 + \tan\phi \cdot \sin\alpha \cdot \cos\alpha/FS} \tag{3.4}$$

where $C_a = c_a \cdot dx/\cos\alpha$ and c_a is the Mohr-Coulomb cohesion intercept.

Since the value of ΔN in equation 3.4 is dependent on a value of the local factor of safety that is exact only at limit equilibrium, slope stability methods inherently calculate incorrect values of the normal stress on the surface on which the factor of safety is computed.

The extent of the critical surface has been assumed to be infinite in the direction perpendicular to the slope cross-section (Figure 3.5). When the lateral extent of the slope is restricted, the critical surface will arc upwards near the lateral boundaries of the slope (Figure 3.6). An approach for accounting for the three-dimensional shape of the limit equilibrium surface on the factor of safety was developed by Chen (1981). His approach consists of the following steps:

1) Locate the critical circular surface for a slope using a slope stability method such as the Simplified Janbu[1] factor of safety.
2) Calculate the factor of safety on the Simplified Janbu critical surface using the Spencer method. This is taken to be the exact value of the two-dimensional factor of safety.
3) Assume that the critical three-dimensional surface is an ellipsoid attached to a cylinder (Figure 3.6) with a cross-section defined by the two-dimensional surface.

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1. The reader should employ the Simplified Janbu option for circles with caution (Boutrup, 1977).

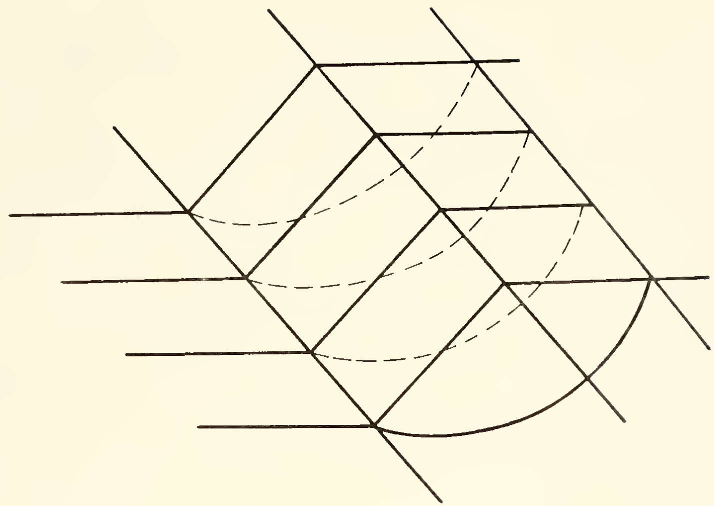


FIGURE 3.5 TWO-DIMENSIONAL TRIAL SURFACE – NO LATERAL LIMITS

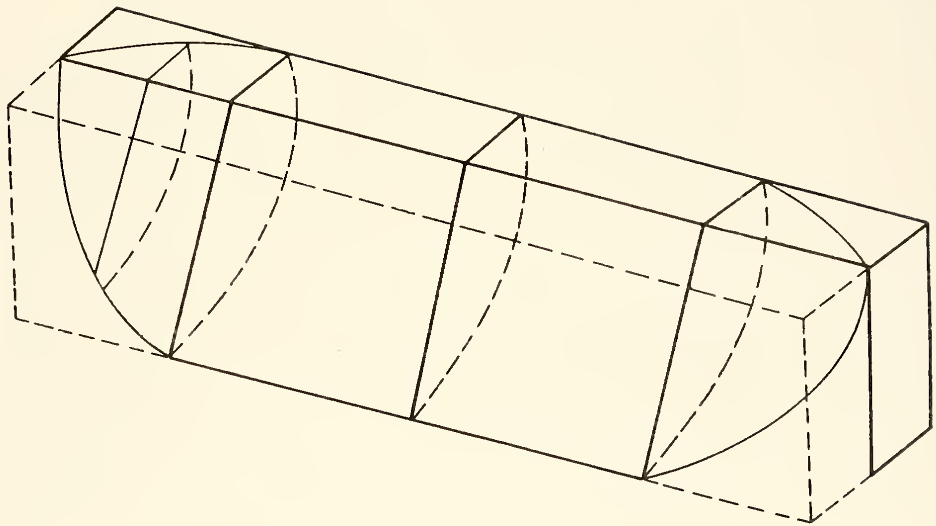


FIGURE 3.6 THREE-DIMENSIONAL VIEW OF CRITICAL SURFACE WITH LATERAL LIMITS

4) Using the LEMIX program (Chen, 1981), calculate the factor of safety on the three-dimensional surface.

This approach will be illustrated with the following example:

<u>Example 3.1</u>

It is desired to assess the three-dimensional factor of safety of the slope shown in Figure 3.7. Assume that the critical three-dimensional surface is an ellipsoid attached to a cylinder (Figure 3.6). The cylinder is defined by the radius of the critical surface for the Simplified Janbu factor of safety which is shown in Figure 3.8. The Simplified Janbu factor of safety on this surface is 1.28. The Spencer factor of safety on the same surface is 1.34. The width of the ellipsoids, l_s, is defined by a specified radius which passes through the endpoint of the cylinder. Using the program LEMIX it is possible to calculate the three-dimensional factor of safety for various values of the l_c/H ratio where

H = height of the slope

l_c = half-width of the cylindrical portion of the sliding mass, i.e., the portion which is identical to the two-dimensional rotational surface (see Figure 3.9).

The results are presented in Table 3.1 and Figure 3.10. As the l_c/H ratio increases, the three-dimensional factor of safety approaches the conventional two-dimensional value.

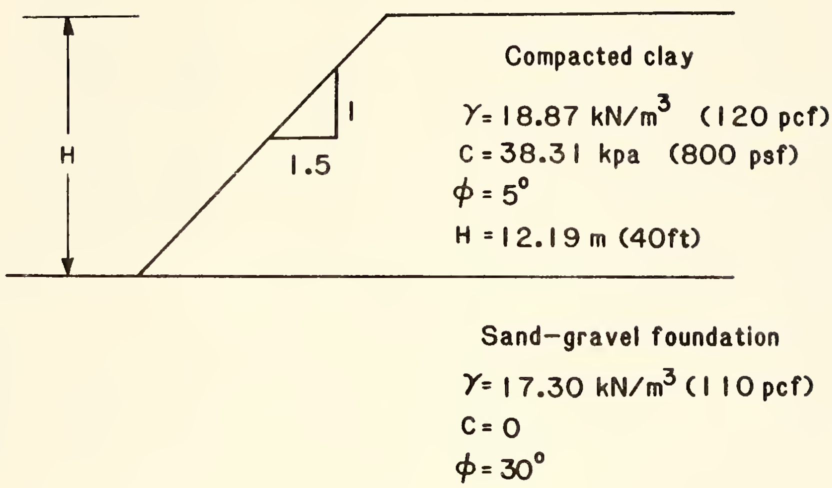
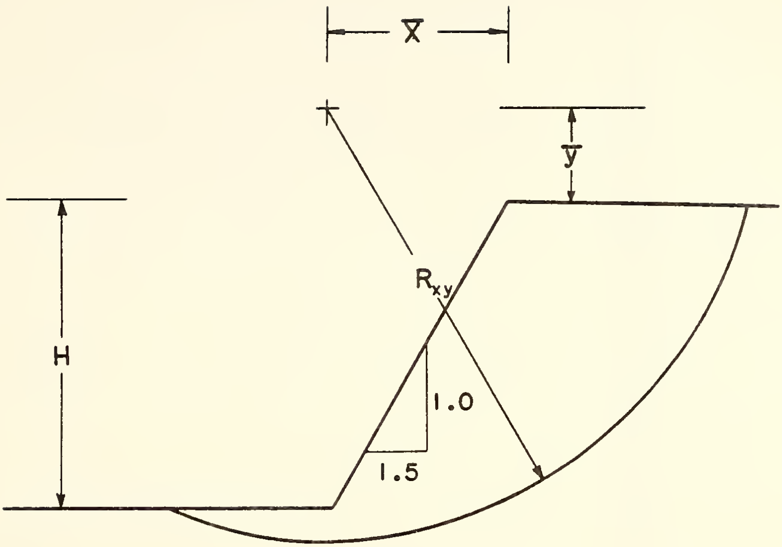


FIGURE 3.7 EMBANKMENT — EXAMPLE 3.1



$\overline{X}$ = 14.63 m (48 ft)
$\overline{y}$ = 14.90 m (48.9 ft)
R_{xy} = 37.42 m (91.6 ft)
H = 12.19 m (40 ft)

FS_{janbu} = 1.28

$FS_{spencer}$ = 1.34

FIGURE 3.8

CRITICAL CIRCLE FOUND WITH SIMPLIFIED JANBU FACTOR OF SAFETY

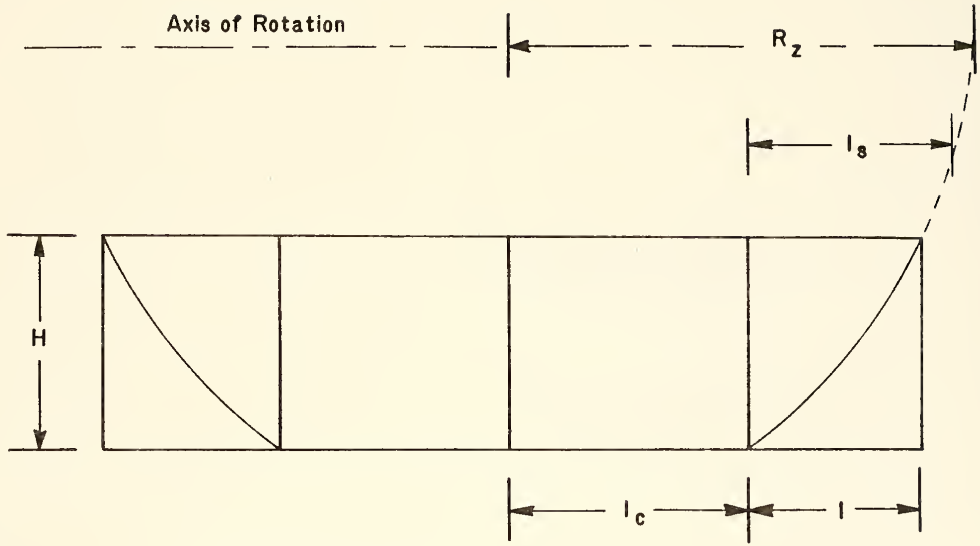


FIGURE 3.9

ELEVATION VIEW OF THREE-DIMENSIONAL CRITICAL SURFACE

Table 3.1 Three-Dimensional Factor of Safety vs. l_c/H ratio - Example 3.1

l_c/H	FS (3-D)
100.00	1.34
50.00	1.34
25.00	1.35
12.50	1.35
6.25	1.37
3.12	1.39
1.56	1.43
0.78	1.50
0.39	1.58
0.19	1.67

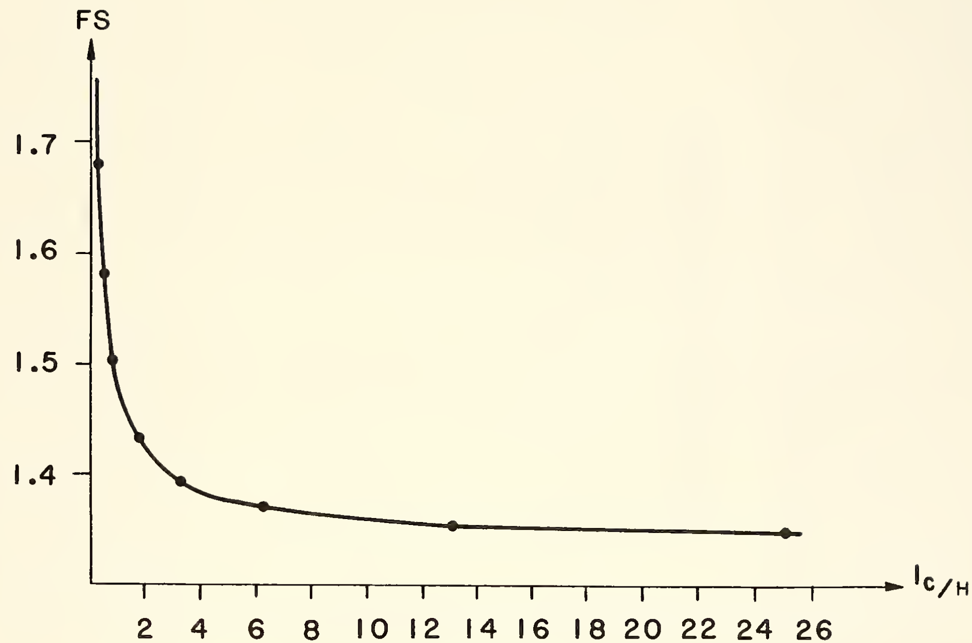


FIGURE 3.10 THREE-DIMENTIONAL FACTOR OF SAFETY OF SLOPE IN FIGURE 3.7 VS. l_c/H

However, at values of l_c/H less than approximately 3.0, the three-dimensional factor of safety is considerably higher than the two-dimensional value. This fact can be taken advantage of in embankments of small width because the most critical surfaces (i.e., those that have a long central cylinder) cannot develop, due to geometrical limitations.

Comments on the STABL Program

STABL is a general purpose slope stability computer program that was developed for the Indiana Department of Highways. It is designed to generate a number of trial surfaces that is specified by the user and to calculate the factor of safety on these surfaces. The user specifies the zone in which he desires the surfaces to be generated. This distinguishes STABL from other slope stability programs with searches specified by central coordinates and radii of circular arcs.

The STABL program does not actually find the critical surface since there is an infinite number of possible random shaped surfaces. Instead, STABL outputs the ten most critical surfaces that are found and their respective factors of safety. These surfaces are plotted automatically. An advantage that results from this methodology is that the factor of safety on various paths through a slope can be compared. For example, consider the embankment in Figure 3.11. In a complete stability analysis, the critical

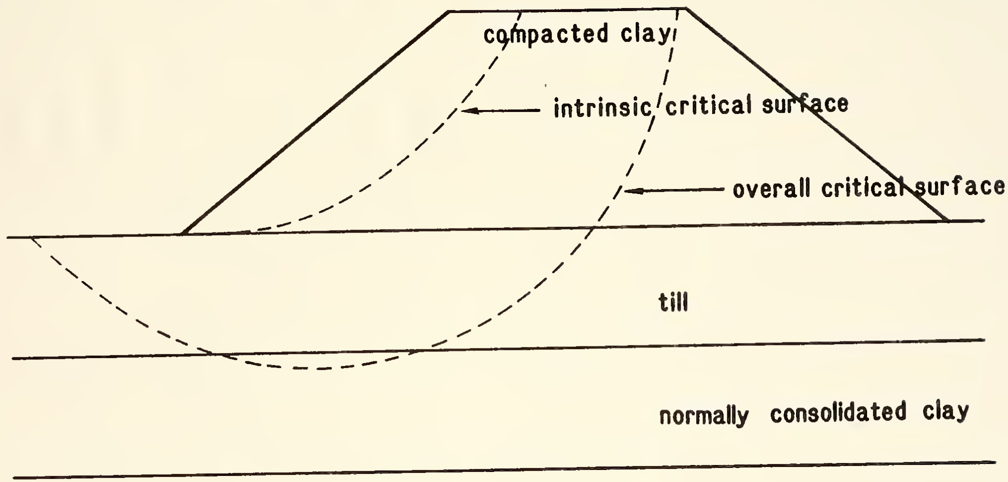


FIGURE 3.11

INTRINSIC AND OVERALL CRITICAL SURFACES OF A SAMPLE EMBANKMENT

surface may be found to pass through till and clay layers that underly the embankment. The corresponding factor of safety is an overall factor of safety. However, if the search is restricted to surfaces that pass exclusively through the embankment itself, the factor of safety on the most critical surface within the embankment is an "intrinsic" factor of safety that is unique to the embankment. The intrinsic factor of safety is frequently an upper bound to the actual factor of safety because surfaces with lower values of the factor of safety may be found through layers that underly the embankment.

STABL has several different surface generation options. A short description of these options is included in the following paragraphs.

The SURFAC option is a command that may be used to determine the factor of safety on a surface of general shape specified by the user. The factor of safety on the surface is calculated by the Simplified Janbu method. The SURFAC option is used to check the factor of safety calculated by STABL against documented solutions.

The SURBIS option is identical to the SURFAC option with the exception that it computes the factor of safety with the Simplified Bishop method. Care should be taken that a circular surface is input because the Simplified Bishop method calculates an incorrect value of the factor of safety

if the coordinates of the limit equilibrium surface are not circular (Bishop, 1955).

The CIRCLE option randomly generates circular surfaces and evaluates the corresponding factors of safety with the Simplified Janbu factor of safety. Searching with circular surfaces generally yields a critical surface whose factor of safety is nearly as low as the factor of safety found on the critical noncircular surface.

The CIRCL2 option is identical to the CIRCLE option except that the factor of safety is calculated by the Simplified Bishop method. This option is generally preferred to the CIRCLE option because there is more experience with the Simplified Bishop method than with the Simplified Janbu method. Also, the Simplified Bishop method does not have the convergence problems sometimes encountered with the Simplified Janbu method for slopes that have high cohesion intercepts and low friction angles.

In nonhomogeneous slopes, the critical surface may be noncircular. The same situation may arise if a homogeneous slope is subjected to a surcharge load or pseudo-static earthquake loading. STABL supplies the RANDOM option for these cases. The RANDOM option pseudo-randomly generates noncircular surfaces and evaluates the factor of safety on them with the Simplified Janbu method. An example of a randomly generated noncircular surface is shown in Figure 3.12.

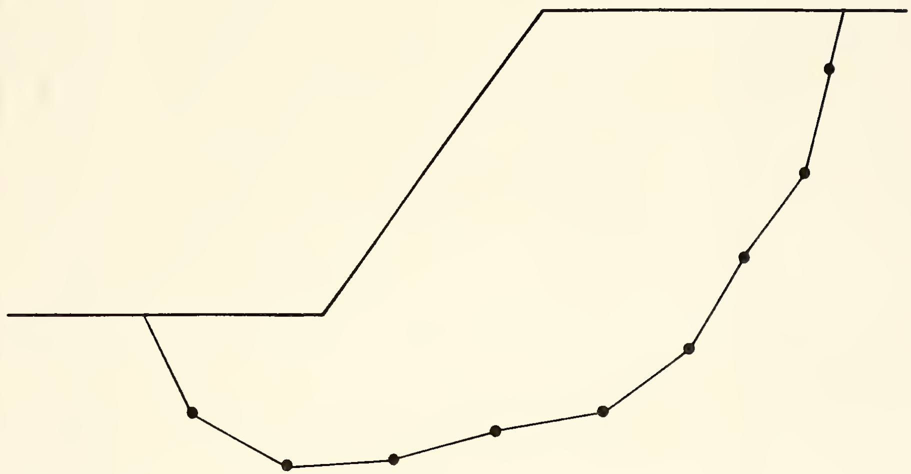


FIGURE 3.12 RANDOMLY GENERATED SURFACE

The BLOCK option specifies a straight line surface between randomly chosen points in boxes of a size specified by the user (Figure 3.13). The remaining portions of these trial surfaces are generated randomly to the left of the leftmost box and to the right of the rightmost box. The Simplified Janbu method is used to calculate the factor of safety on these surfaces. The BLOCK option is especially effective when there is a weak seam or bedding plane in the slope.

Using a method of slices for the analysis of block shaped surfaces is consistent with the factor of safety used for surfaces of other shapes because it imposes the condition that the local factor of safety and the global factor of safety are equal everywhere along the trial surface. This is different than ordinary methods of calculating the factor of safety of a block shaped surface. These methods implicitly assume that the local factor of safety on the passive wedge and the active wedge is unity although the factor of safety on the central block must be greater than unity for stability.

The BLOCK2 option is identical to the BLOCK option with the exception that the portions of the trial surface to the left of the leftmost box and to the right of the rightmost box are generated to simulate Rankine active and passive zones respectively. This option was developed because the BLOCK option yields a minimum factor of safety on surfaces

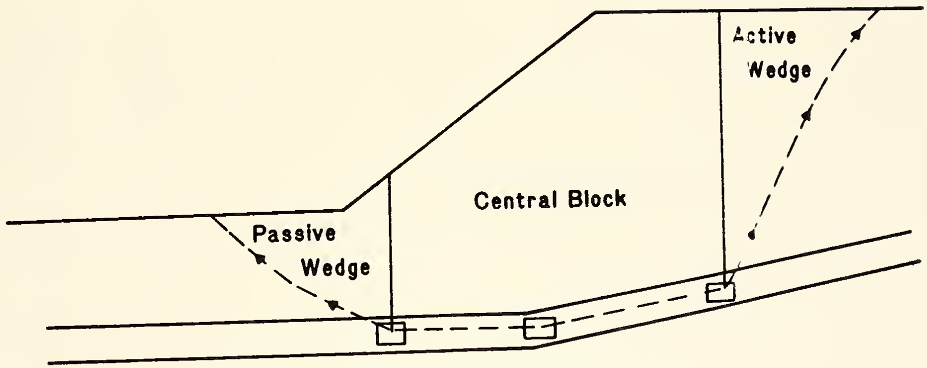


FIGURE 3.13 GENERATION OF A BLOCK SHAPED SURFACE

that approximate the Rankine state. Therefore, use of the BLOCK2 option permits a savings in computational effort. The BLOCK option has been retained, however, for use in heterogeneous soil profiles.

While providing advice on the use of STABL in the capacity of STABL consultant at Purdue, the author noted that certain problems arose repeatedly. These problems included:

(1) surfaces with unacceptable shapes

(2) the direction of surface generation

(3) direction limits on benched slopes

(4) variability of results due to the random generation of surfaces

(5) a lack of experience with the Simplified Janbu factor of safety.

These problems are discussed in the following paragraphs.

Occasionally, when the RANDOM option is used, one or more of the ten most critical surfaces that are generated will be concave or even have a reverse curvature as shown in Figure 3.14. Such surfaces are kinematically impossible. Therefore, the factors of safety computed on these surfaces should be disregarded.

STABL assumes that a slope rises from left to right. Therefore, it generates surfaces that progress from left to right (Figure 3.15a). If the user attempts to input a slope that rises from right to left, STABL would continue to generate

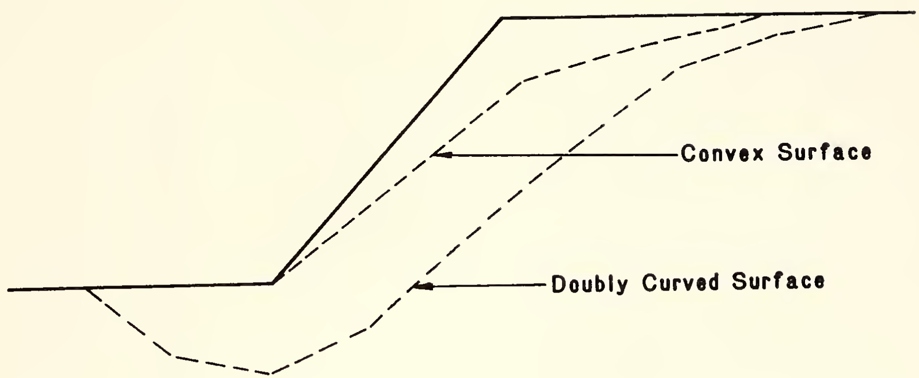
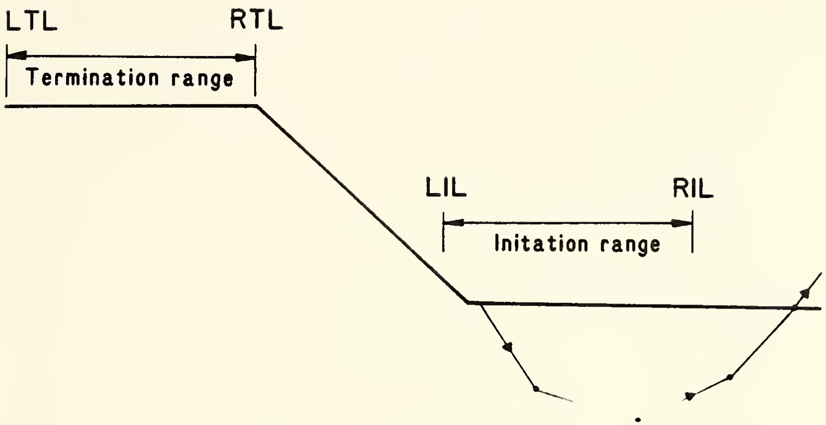


FIGURE 3.14 KINEMATIALLY IMPOSSIBLE SURFACES



LIL leftmost initation limit
RIL rightmost initation limit
LTL leftmost termination limit
RTL rightmost termination limit

FIGURE 3.15a EXAMPLE OF SLOPE INPUT BACKWARDS

trial surfaces from left to right, which never reach the termination range. In this case, STABL outputs error message RC-06 and halts execution of the problem. This error can be avoided simply by inputting the data so that the slope rises from left to right (Figure 3.15b).

When a user desires the initiation limits to straddle a break in the ground surface where the inclination of the slope decreases from left to right, the direction limits must be compatible with each of the segments that lie between the leftmost initiation limit and the rightmost initiation limit. Otherwise, a situation may develop where STABL will generate a trial surface that goes outside the slope (see Figure 3.16). If the trial surface does not cross the ground surface between the termination limits, error message RC-10 is output and execution of the problem is halted. If the trial surface does cross the ground surface between the termination limits, the results are meaningless, although no error is detected by the program. To avoid this difficulty, the user should define left and right initiation points for each segment of the ground surface in the initiation range. Each set of initiation limits should be executed with appropriate direction limits on a separate run. This is illustrated in Figure 3.17.

The inclination of the first two segments of circular and random shaped surfaces generated by STABL and the inclination of subsequent segments of random shaped surfaces are

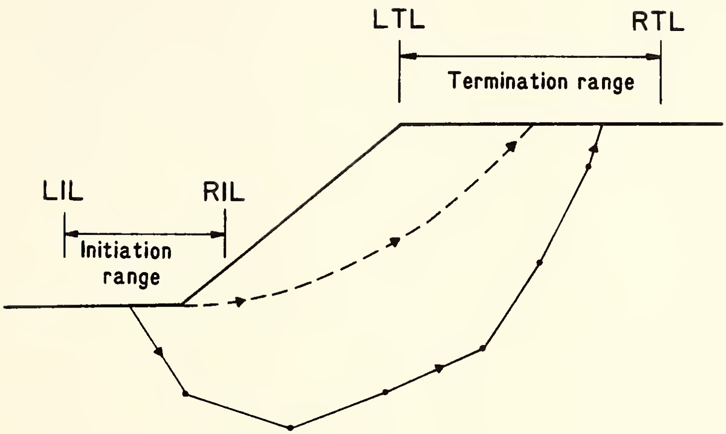


FIGURE 3.15b

EXAMPLE OF CORRECTLY INPUT SLOPE

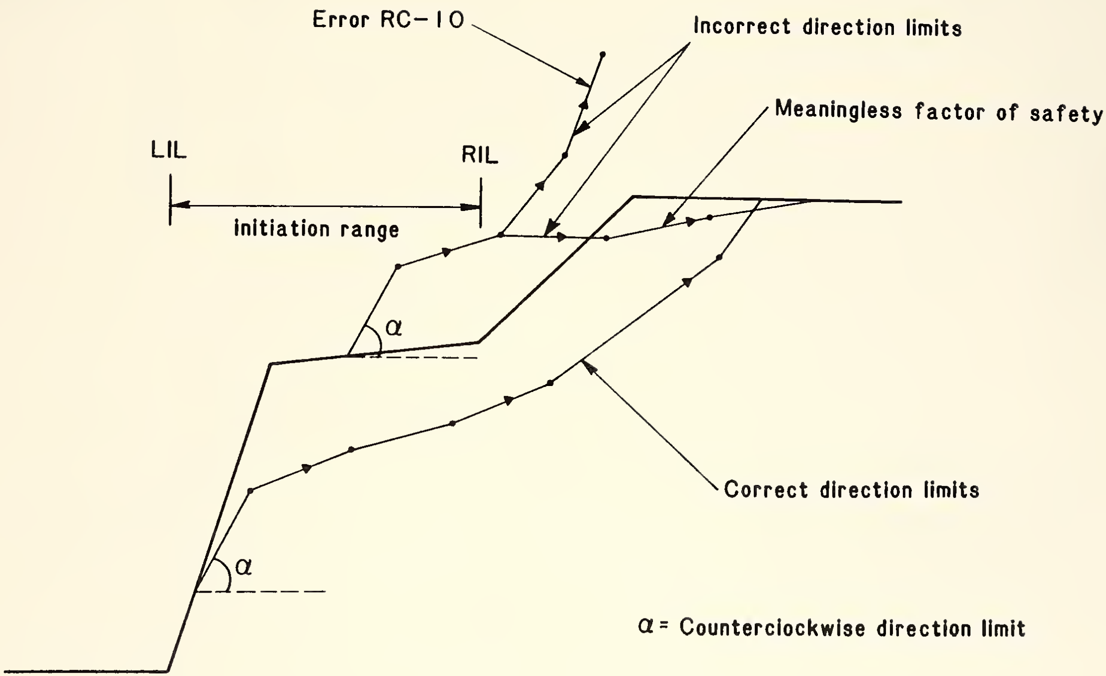


FIGURE 3.16 ERRORS POSSIBLE DUE TO INCORRECT DIRECTION LIMITS ON BENCHED SLOPES

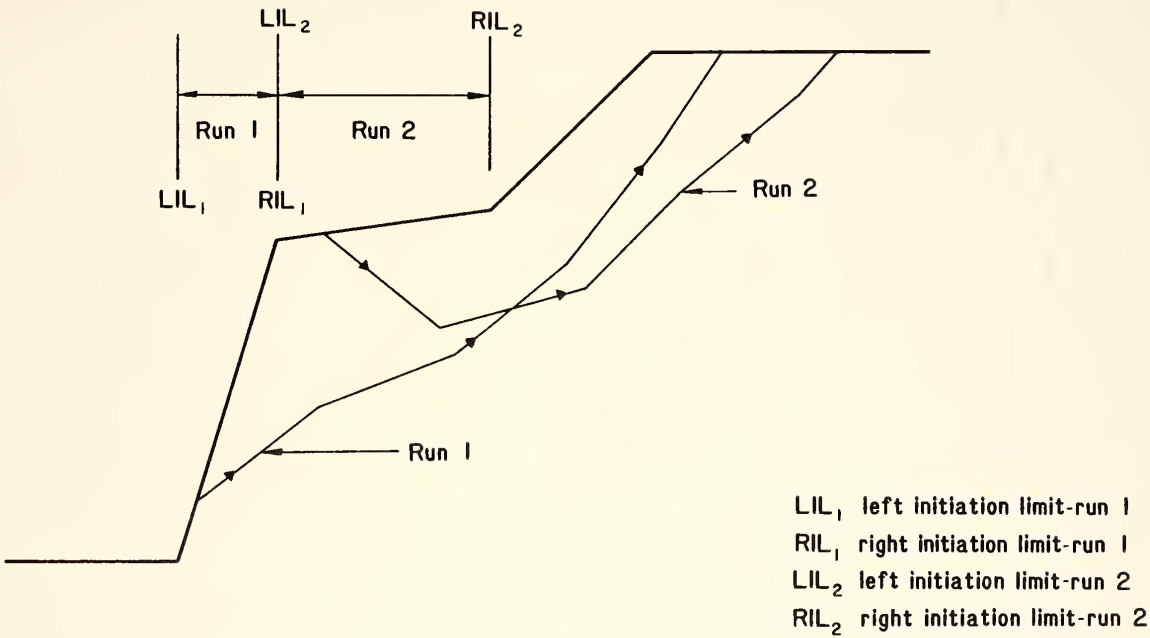


FIGURE 3.17 EXAMPLE OF CORRECT INITIATION LIMITS FOR A BENCHED SLOPE

pseudo-randomly generated with the aid of a random number generator called RANF. RANF is a computer supplied function on CDC 6000 series computers that generates uniformly distributed random numbers between 0 and 1. However, since the generation sequence starts with the same built-in seed each time that the program is used, it always yields identical random number sequences. Therefore, STABL creates identical surfaces each time it is run on a CDC computer unless the seed is modified. When STABL is run on an IBM computer, the user must supply a random number generator. Some users have employed generators that yield a different sequence each time the program is run. When this is the case, STABL will create different trial surfaces each time the program is run, and consequently, slightly different values of the factor of safety.

Originally, all the STABL options for calculating the factor of safety on noncircular surfaces employed the Carter method (Carter, 1971). This method makes the following assumptions:

1) All interslice forces are equal in magnitude and opposite in sign. Therefore, they may be neglected in the analysis of the factor of safety.
2) The minimum factor of safety on any trial surface is obtained when moments of the driving and resisting forces are taken about a point that is at an infinite height above the slope.

3) All other assumptions are identical to those made by the Simplified Bishop method.

The expression for the factor of safety given by the Carter method does not require the calculation of interslice forces. Therefore, the reasonableness of the line of thrust need not be checked as is required by methods such as the extended Spencer method (Spencer, 1973) and the Rigorous Bishop method (Bishop, 1955). This permits the Carter factor of safety to be calculated with relatively little computational effort.

Boutrup (1977) indicated that the Carter method is identical to the Rigorous Janbu method (Janbu, 1954) with the simplifying assumption that interslice forces may be neglected from the formulation. This implies that the Carter method (or the Simplified Janbu method) does not satisfy the requirements of statics for each slice, although horizontal force equilibrium is satisfied for the soil mass above a trial surface. Experience with other incomplete equilibrium formulations such as the Fellenius method of slices indicates that incomplete equilibrium techniques give values of the factor of safety that are lower than those found by complete equilibrium techniques. Since the Simplified Janbu method is conservative compared to more exact methods, it may be worthwhile to adjust it to correspond to methods that yield results that are closer to complete equilibrium solutions.

In order to provide a guide for performing this adjustment, values of the factor of safety have been computed for simple homogeneous slopes with various values of the sideslope, β, and the parameter, $\lambda = \tan\phi/[c/\gamma H]$ for both the Friction Circle method and the Simplified Bishop method. The differences (in percent) of the factor of safety of these two methods relative to the Simplified Janbu factor of safety using circular surfaces were also computed. The results are given in Table 3.2 and Figures 3.18 and 3.19. The Simplified Janbu factor of safety may be adjusted to simulate these familiar methods of determining the factor of safety with the following expression:

$$FS = FS_{STABL} \cdot [1.0 + \%\ error/100] \qquad (3.5)$$

where

FS = the value of the factor of safety that has been adjusted to simulate another method.

FS_{STABL} = the minimum factor of safety obtained with the Simplified Janbu method coded in the STABL program.

$$\%\ error = \frac{FS_{(desired\ method)} - FS_{STABL}}{FS_{STABL}} \times 100$$

Values of % error may be interpolated from Figures 3.18 and 3.19 for the Friction Circle and the Simplified Bishop methods, respectively. This adjustment was developed for simple, unsaturated, homogeneous slopes with circular trial

Table 3.2 Factor of Safety for Values of Sideslope and λ. (after Boutrup, 1977)

Method	λ	$\cot\beta$ 1.5	2.5	3.5
1	2	1.147	1.443	1.714
2		1.103	1.351	1.595
% error		3.99	6.81	7.46
1	5	1.785	2.409	3.008
2		1.703	2.277	2.850
% error		4.82	5.80	5.54
1	8	2.368	3.309	4.226
2		2.262	3.154	4.045
% error		4.69	4.91	4.47
1	20	2.257	3.347	4.425
2		2.167	3.242	4.307
% error		3.72	3.24	2.74
1	50	1.902	2.948	3.991
2		1.855	2.893	3.928
% error		2.53	1.90	1.60

Method 1 - Friction Circle
Method 2 - STABL2, Simplified Janbu
$\lambda = \tan\phi/[c/(\gamma H)]$

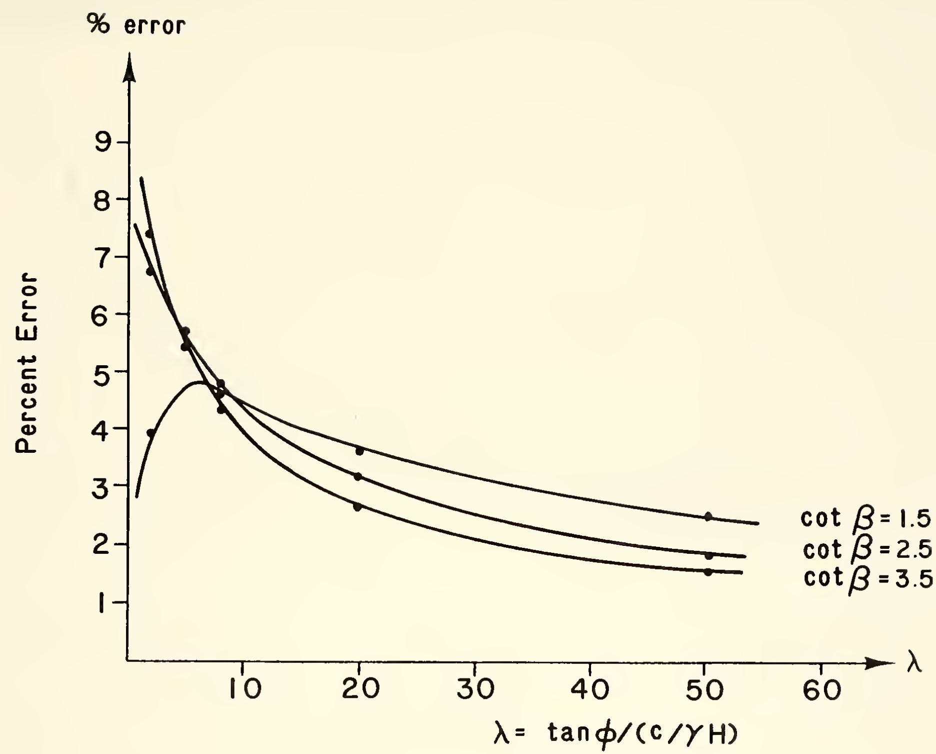


FIGURE 3.18 COMPARISON OF STABLS SIMPLIFIED JANBU FACTOR OF SAFETY WITH THE FRICTION CIRCLE METHOD

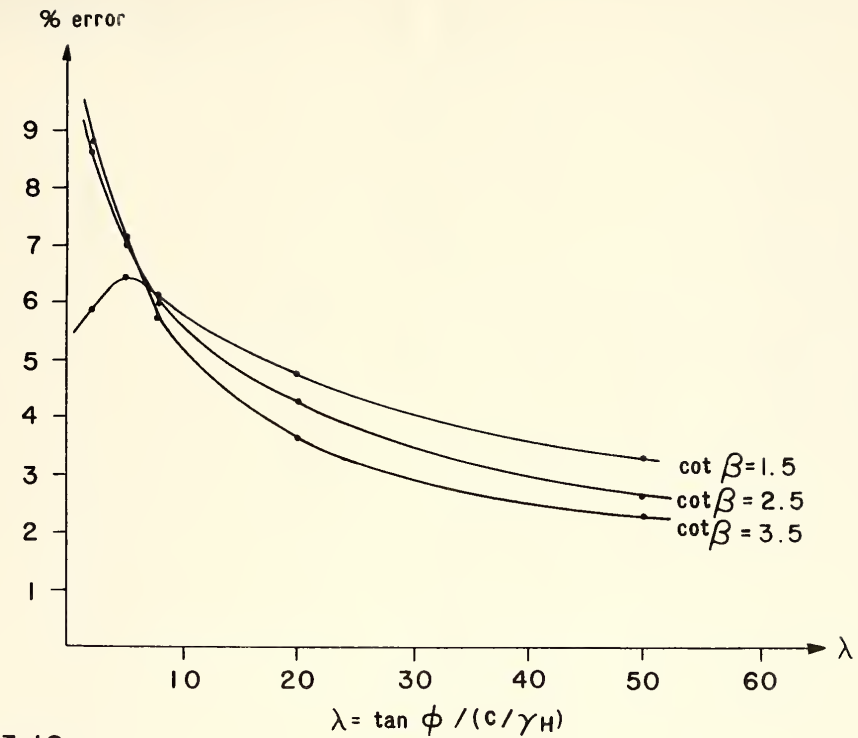


FIGURE 3.19

COMPARISON OF STABL'S SIMPLIFIED JANBU FACTOR OF SAFETY WITH THE SIMPLIFIED BISHOP METHOD

surfaces. However, it may be used approximately on slopes that do not satisfy these conditions.

It should be emphasized that this adjustment is only an approximate procedure to correct an incomplete equilibrium technique, i.e., a method that does not satisfy equilibrium of the slices above the trial limit equilibrium surface. It would be more consistent to use a method of calculating the factor of safety that satisfies equilibrium of each of the slices.

Corrections to STABL

Originally, the STABL program calculated the Simplified Bishop factor of safety by multiplying the terms of the following summation which is used for calculating the Simplified Janbu factor of safety

$$\sum_{i=1}^{n} \left[\frac{A_1 + FS \cdot A_2}{FS + A_3}\right] = 0 \qquad (3.6)$$

by $\bar{y} = R \cdot \cos\alpha$ (see Boutrup, 1977)

where

$\bar{y}$ = vertical distance to the bottom of the slice from the moment center

R = radius of the trial circle

α = slope of the bottom of the slice

n = number of slices

A_1, A_2, A_3 = terms reflecting the conditions of slice (see Siegel, 1975).

FS = the factor of safety

Since the value of R is constant for circular surfaces, this reduces to:

$$\sum_{i=1}^{n} [\cos\alpha \frac{A_1 + FS \cdot A_2}{FS + A_3}] = 0 \tag{3.7}$$

This procedure is very efficient for computer coding because the terms of the summation for the Simplified Bishop method may be obtained by multiplying the terms of the Simplified Janbu summation by their respective values of cos α.

Unfortunately, this formulation assumes that all forces acting on a slice act along the base of the slice (Howland, 1982). This is not true if pseudo-static earthquake forces or boundary surcharge forces act on a slice or if the water table extends above a slice. Therefore, STABL gave incorrect values of the Simplified Bishop factor of safety in these circumstances. To rectify this problem, the author recoded the Simplified Bishop factor of safety to include the differences in the moment arms of these forces. The expression for the factor of safety is (details of the derivation are given in Appendix A):

$$FS = \frac{\sum_{1}^{n} [\frac{A_1}{1 + A_2/FS}]}{\sum_{1}^{n} A_3 - \sum_{1}^{n} A_4 + \sum_{1}^{n} A_5} \tag{3.8}$$

where

FS = factor of safety

$A_1 = C'_a + \tan\phi'_a \sec\alpha(\Delta W(1-k_v) + \Delta Q\cos\delta + \Delta U_\beta + \cos\beta - \Delta U_\alpha \cos\alpha)$ (3.9a)

$A_2 = \tan\phi'_a \cdot \tan\alpha$ (3.9b)

$A_3 = (\Delta W(1-k_v) + \Delta U_\beta \cdot \cos\beta + \Delta Q \cdot \cos\delta) \sin\alpha$ (3.9c)

$A_4 = (\Delta U_\beta \cdot \sin\beta + \Delta Q \cdot \sin\delta)(\cos\alpha - h/R)$ (3.9d)

$A_5 = k_h \cdot \Delta Q \; (\cos\alpha - h_{eq}/R)$ (3.9e)

These A-terms are not the same as the terms used for determination of the Simplified Janbu factor of safety, although the variables in equations 3.9a through 3.9e are identical to the variables used when the program was originally developed (Siegel, 1975). The factor of safety must be calculated with an iterative procedure because equation 3.8 is implicit. In order to make the solution technique for the Simplified Janbu method analogous to that of the Simplified Bishop method, the Newton-Raphson method which was used for the Simplified Janbu factor of safety (Siegel, 1975) was replaced with an implicit iterative technique. The implicit formulation of the factor of safety is believed to be more efficient in achieving a convergence on the value of the factor of safety when incomplete equilibrium analyses are performed, especially when the factor of safety is much greater or much less than the value that is initially assumed (Boutrup, 1977).

Convergence of the iteration scheme is achieved if the assumed and back-calculated values of the factor of safety on a surface differ by less than 0.005. The maximum number of iterations is limited to ten. If the solution does not converge, the coordinates of the surface and the last back-calculated value of the factor of safety are output. The editing that is required to incorporate these changes into the STABL code is listed in Appendix A.

Strength Parameters of Compacted Clays

The As-Constructed Condition

To replicate the condition existing at the end of construction, a clay sample compacted to simulate field compactive effort must be tested according to UU procedures. This means that the sample is loaded quickly so that there is no time for the excess pore pressure induced by the loading to dissipate.

Ideally, the loading should be performed to simulate the stress path that the soil undergoes in the field. Also, the loading should reflect the rotation in the direction of the stresses that occurs in the embankment. Unfortunately, the initial stresses and stress changes in an embankment are not amenable to calculation. Even if this were possible, it would be necessary to perform a prohibitive number of tests along various stress paths in order to model the strength of

the soil as a function of position in the embankment. Consequently, the current practice is to simulate the UU shear strength of unsaturated, compacted soil under an ordinary triaxial loading. It has been shown that unsaturated, compacted clays tested in this fashion have a strength line that is uniquely defined by their water content and the compactive effort they have been subjected to (Weitzel, 1979). The shear strength data for St. Croix clay are given in Table 3.3 and in Figure 3.20. These data may be represented as linear strength lines by performing linear regressions on the data in Table 3.3. The Mohr-Coulomb parameters may be obtained from the strength lines through simple trigonometric relationships (Holtz and Kovacs, 1981). The results of the regression are given in Table 3.4 and Figures 3.21 and 3.22. The correlation coefficient, r, decreases as the water content of the soil increases. Even so, the correlations are relatively high except when the water content of the soil is well above the OMC. Using Figure 3.21 and 3.22 it is possible to estimate the UU Mohr-Coulomb strength parameters of St. Croix clay at any water content for the compaction level at which the soil samples were prepared.

Example 3.2

It is desired to calculate the intrinsic factor of safety of the St. Croix clay embankment shown in Figure 3.23. The water content of the soil is 19% and the

Table 3.3 UU Shear Strength of Compacted St. Croix Clay (after Weitzel, 1979)

Compaction	w%	$(\sigma_1-\sigma_3)_f/2$ (kPa)	σ_3 (kPa)	$(\sigma_1+\sigma_3)_f/2$ (kPa)
Low Energy*	20.75	132	0	132
		115	0	115
		208	138	346
		229	276	505
		283	276	559
		249	414	663
		322	414	736
	22	91	0	91
		104	0	104
		179	138	317
		197	138	335
		192	276	468
		226	414	640
	24	97	0	97
		79	0	79
		118	138	256
		139	138	277
		148	276	424
		135	414	549
		133	414	547
	27	56	0	56
		52	0	52
		73	138	211
		59	138	197
		73	276	349
		83	276	359
		66	414	480
		67	414	471

* Low Energy compaction has 60% of the compactive effort of standard compaction.

Table 3.3 (continued)

Compaction	w%	$(\sigma_1-\sigma_3)_f/2$ (kPa)	σ_3 (kPa)	$(\sigma_1+\sigma_3)_f/2$ (kPA)
Standard	19	164	0	164
		146	0	146
		262	138	400
		294	138	432
		381	276	657
		422	414	836
	20.1	212	0	212
		191	0	191
		308	138	446
		320	276	596
		352	276	628
		372	414	786
	21.6	200	0	200
		158	0	158
		188	138	326
		237	276	513
		250	276	526
		243	414	657
		240	414	654
	25.0	74	0	74
		73	0	73
		115	138	253
		87	138	225
		89	276	365
		107	276	383
		150	414	564
		99	414	513

Table 3.3 (continued)

Compaction	w%	$(\sigma_1-\sigma_3)_f/2$ (kPa)	σ_3 (kPa)	$(\sigma_1+\sigma_3)_f/2$ (kPa)
Modified	13.8	869	138	1007
		828	138	966
		1007	276	1283
		942	276	1218
		1044	414	1458
	15.0	661	0	661
		716	0	716
		736	138	874
		885	276	1161
		977	276	1253
		1033	414	1447
		915	414	1329
	16.3	608	0	608
		667	0	667
		720	138	858
		744	138	882
		866	276	1142
		774	276	1050
		747	414	1161
	19.0	449	0	449
		497	138	635
		417	138	555
		460	276	736
		455	276	731
		479	414	893

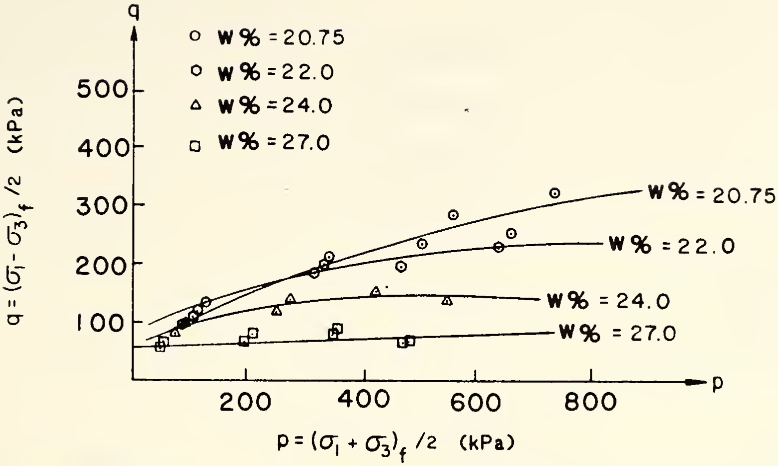


FIGURE 3.20a UU STRENGTH LINES OF ST. CROIX CLAY-LOW ENERGY COMPACTION

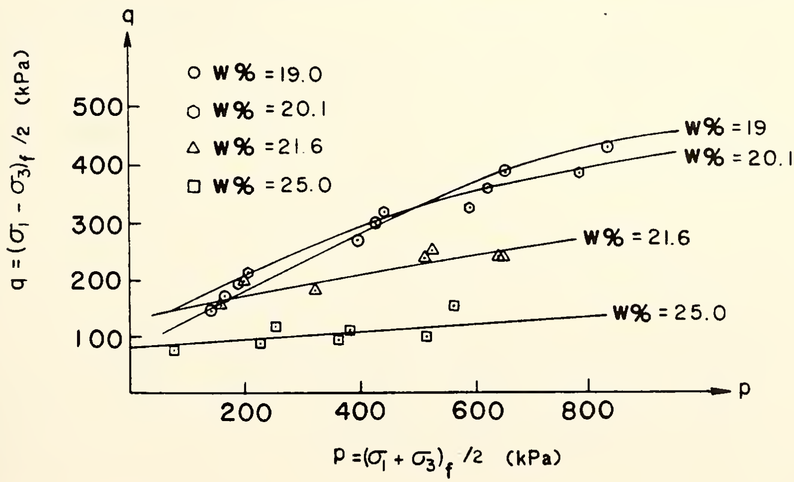


FIGURE 3.20b UU STRENGTH LINES OF ST. CROIX CLAY-STANDARD COMPACTION

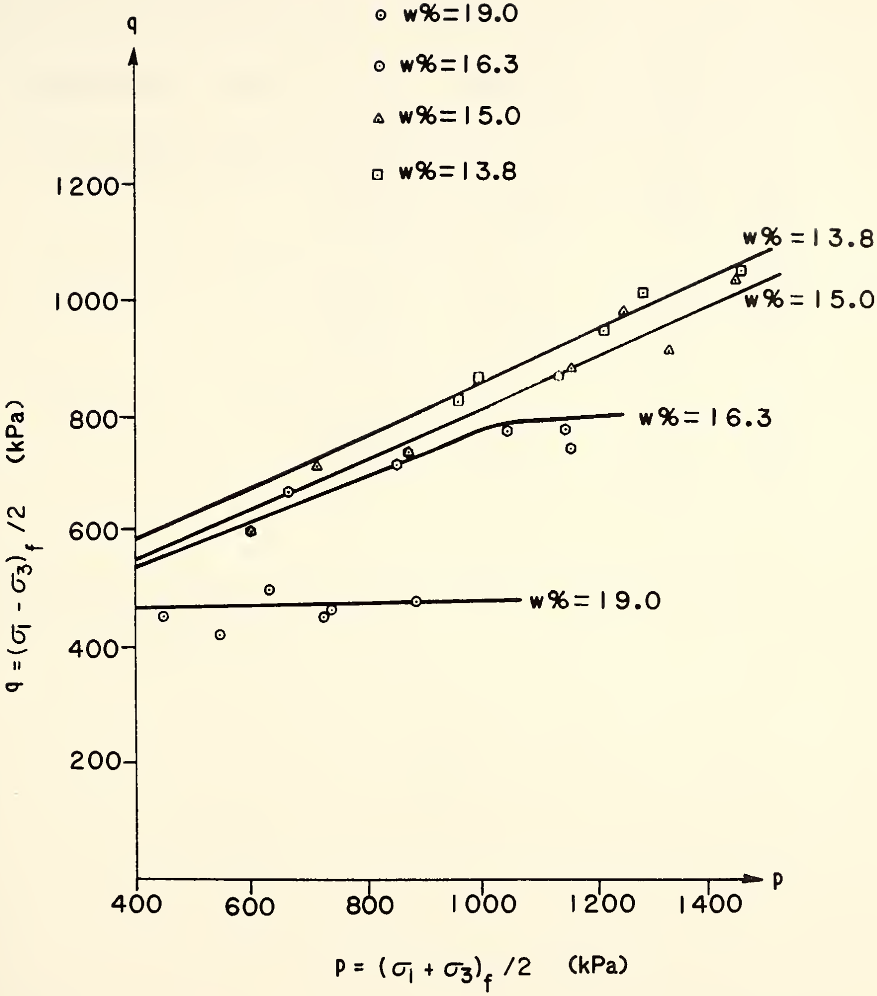


FIGURE 3.20c

UU STRENGTH LINES OF ST. CROIX CLAY–
MODIFIED COMPACTION

Table 3.4 UU Mohr-Coulomb Parameters of Unsaturated St. Croix Clay

Compaction	OMC	w%	ϕ (degrees)	c (kPa)	r
Low Energy*	24.0	20.75	17.1	95.8	.96
		22.0	14.0	83.6	.94
		24.0	5.9	88.8	.81
		27.0	2.2	55.9	.62
Standard	21.6	19.0	24.1	108.5	.99
		20.1	17.7	155.1	.98
		21.6	8.8	152.4	.90
		25.0	6.1	67.0	.78
Modified	16.3	13.8	26.0	465.4	.98
		15.0	26.5	416.1	.97
		16.3	19.0	461.8	.88
		19.0	4.7	405.8	.47

*Low Energy compaction has 60% of the compactive effort of Standard compaction.

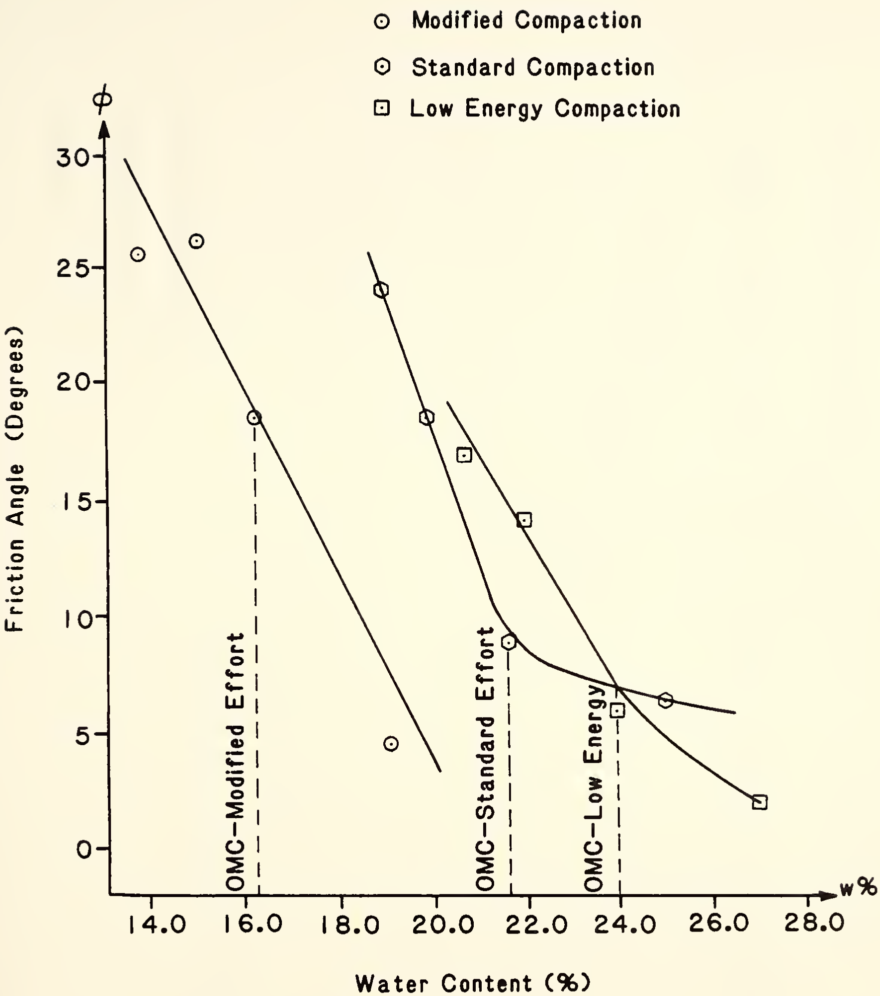


FIGURE 3.21 THE VARIATION OF ϕ OF COMPACTED ST. CROIX CLAY VS. WATER CONTENT

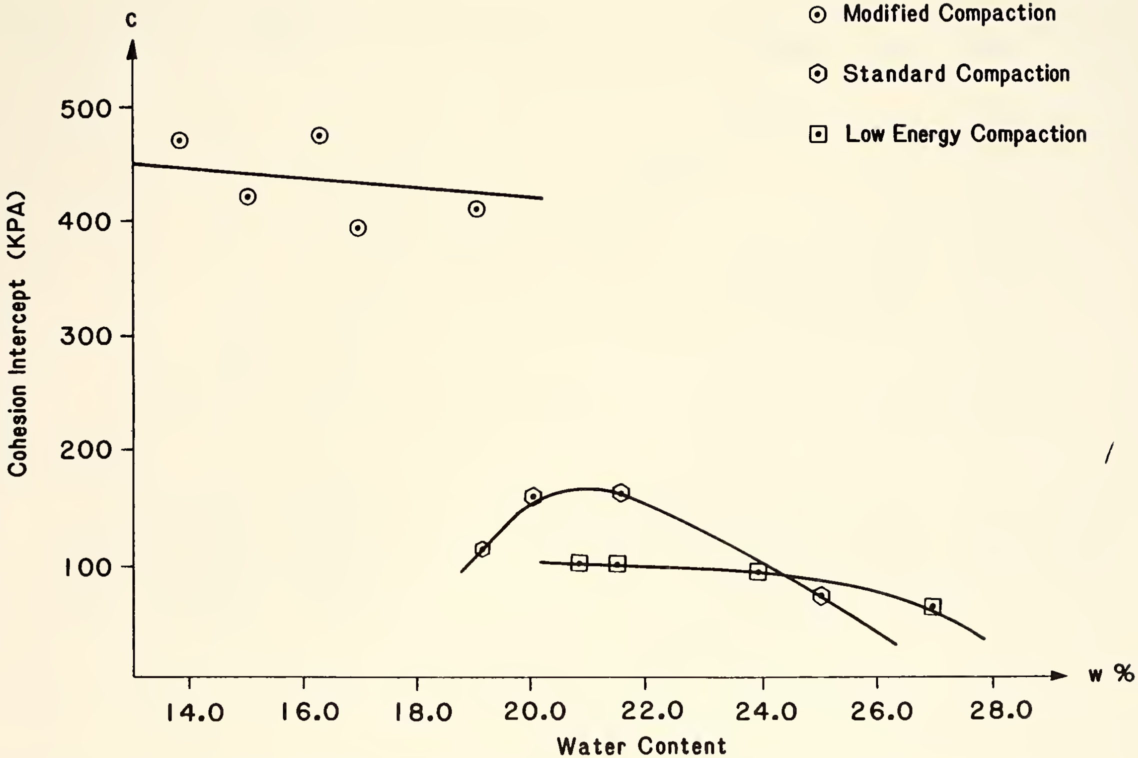
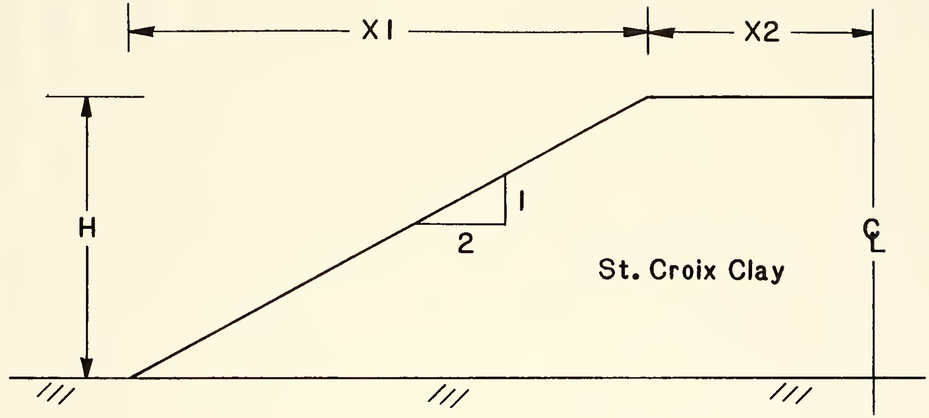


FIGURE 3.22 THE VARIATION OF THE COHESION INTERCEPT OF COMPACTED ST. CROIX CLAY VS. WATER CONTENT



$H = 30.48$ m (100 ft)
$X1 = 60.96$ m (200 ft)
$X2 = 6.10$ m (20 ft)

$\gamma_m = 19.02$ kN/m³ (121 pcf)
$c = 108.5$ kPa (2266 psf)
$\phi = 24.1^{\circ}$
W% = 19

FIGURE 3.23 SLOPE – EXAMPLE 3.2

compactive effort is Standard. It can be seen from Figures 3.21 and 3.22 that ϕ and c are 24.1° and 108.5 kPa (2266 psf) respectively. The moist density of St. Croix clay at this water content and compactive effort is 19.04 kN/m^3 (121 pcf). Using STABL, the minimum factor of safety obtained on a random shaped surface through the embankment is 2.96.

The Long Term Condition

To replicate the behavior of an embankment long after it has been constructed, it is necessary to adjust strength test procedures. Long term conditions are bounded by two conditions. The first extreme is that long after construction the embankment remains unsaturated. The proper way to run a test on the soil for this situation is the consolidated-drained (CD) test. The soil is consolidated to the expected state of stress in the embankment and then sheared at a rate that is sufficiently slow to allow any excess pore pressure developed by the loading to dissipate. Ordinarily, the requirement that the soil should be consolidated to the expected state of stress is relaxed. Instead, the soil is isotropically consolidated to several different confining pressures and subsequently sheared. Experience indicates that this has little effect on the strength envelope that is obtained for many soils. The CD test is not performed on a routine basis because it must be run very slowly to allow excess pore pressures to dissipate.

The second extreme condition is that of the embankment becoming completely saturated sometime after construction. This case, like the case of the sudden drawdown of water behind a dam, requires the use of a consolidated-undrained (CU) test. The soil is consolidated to the expected state of stress in the field and then sheared quickly. As the soil is sheared, the excess pore pressure due to the stress changes is measured. As was the case with CD parameters, the requirement of consolidating the sample to the insitu state of stress is usually relaxed.

A major problem involved in analysis of the second extreme condition is the development of excess pore pressure. The excess pore pressure that develops in the laboratory is caused by the changes in the stresses that are applied to the sample. The excess pore pressure in the field is caused by the increase of density due to saturation and the gradual stress changes caused by displacements within the embankment that arise from changes from the UU to CU or CD conditions. Therefore, the excess pore pressures measured in a triaxial test cannot be used to predict excess pore pressures in an embankment.

An approximate method can be used to insure that the excess pore pressures will be zero or negative. It has been shown that the pore pressure parameter, A_f, which relates excess pore pressure and deviator stress, varies with the overconsolidation ratio (Henkel, 1956). Typical results are

shown in Figure 3.24. The overconsolidation ratio (OCR) of a compacted clay may be estimated with the ratio of the compactive prestress to the overburden pressure. Therefore, the depth in the embankment above which positive pore pressure can not develop regardless of stress change (i.e., $A_f \leq 0$) is:

$$H = \frac{P_s}{OCR_o} \gamma_m \qquad (3.10)$$

where

H = depth beneath embankment surface to which $u \leq 0$

u = excess pore pressure due to the stress change

P_s = compactive prestress

γ_m = moist density of the soil

OCR_o = the OCR at which $u \leq 0$

Results in Figure 3.24 indicate that OCR_o will vary between 4 and 5 for a natural saturated clay. It is reasonable to assume that the same is true for compacted clays. Typical results for an embankment built of compacted St. Croix clay are given in Table 3.5. These results assume that OCR_o = 4.0. Values of P_s and γ_m are taken from DiBernardo (1979).

Actually, the excess pore pressure caused by shear of compacted clays at strains smaller than failure strains can be larger than is predicted by use of the pore pressure parameter in Figure 3.24. Therefore, OCR_o will not

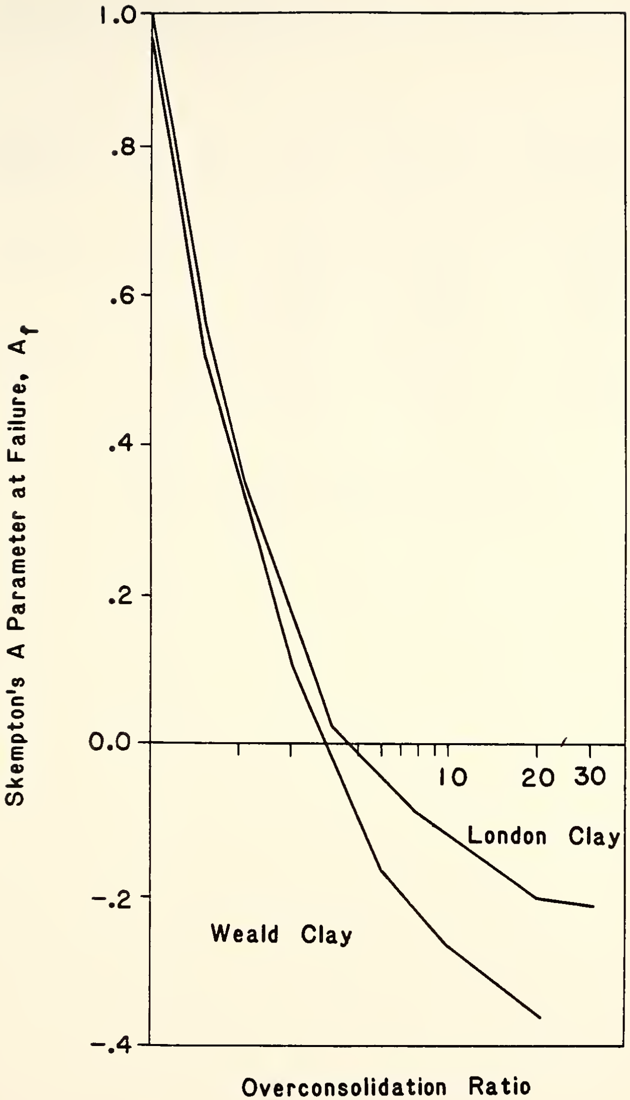


FIGURE 3.24 A_f VS. OVERCONSOLIDATION RATIO
(AFTER HENKEL, 1956)

Table 3.5 Depth to Zone of Positive Pore Pressure Development

Compactive Effort	OMC(%)	w(%)	P_s(kPa)	γ_m(kN/m^3)	H* (meters)
Standard	22.0	19.5	430.6	18.70	5.76
		22.0	271.8	19.51	3.48
		25.0	68.6	19.42	.88
Modified	16.5	14	1166.3	20.17	14.47
		16.5	717.9	20.97	8.56
		19.5	212.7	20.63	2.58

*H = depth to the bottom of the zone of positive pore pressure development

correspond exactly to a state of zero excess pore pressure generation. In any event, the depth above which the excess pore pressure is taken to be less than zero will sometimes be only a small portion of the embankment height. Therefore, stress changes will cause an increase in pore pressures. Unfortunately, the stress changes and hence the pore pressure changes can not be predicted easily. Consequently,, the only convenient approach is to assume that pore pressure in an embankment is equal to the head of water above the location in question.

The CU friction angle and cohesion intercept of laboratory compacted and saturated St. Croix clay are approximately 20° and 15 kPa regardless of compaction variables or stress path (Johnson, 1979). Therefore, it is possible to develop a graph of the intrinsic factor of safety for every possible geometry. For the unsaturated case, the moist density of the embankment soil is used. For the saturated case, the saturated density of the embankment soil is used and a water table is assumed to run along the free surface of the embankment. In these examples the effective stress parameters are assumed to be the same for both saturated and unsaturated compacted clay. The results are given in Table 3.6 and Figures 3.25 and 3.26. These results were developed using the Simplified Janbu factor of safety on circular surfaces that were restricted to remain above the elevation of the toe and to exit the slope less than 10 meters from the crest of the slope.

Table 3.6 Intrinsic Simplified Janbu Factors of Safety of Compacted St. Croix Clay Embankments Using Laboratory Compacted CU Shear Strength

Unsaturated Case

H	β	FS
5m	30°	2.12
	35	1.86
	40	1.71
	45	1.61
	50	1.52
	55	1.47
	60	1.37
	65	1.30
	70	1.24
10m	20°	2.12
	25	1.70
	30	1.44
	35	1.26
	40	1.16
	45	1.08
	50	1.02
	55	0.95
20m	17.5	1.83
	20.0	1.59
	22.5	1.43
	25.0	1.29
	30.0	1.08
	32.5	1.01
30m	15.0	1.90
	17.5	1.62
	20.0	1.42
	25.0	1.14
	30.0	0.96
40m	15.0	1.78
	20.0	1.32
	25.0	1.06
	30.0	0.90

Table 3.6 (continued)

Saturated Case

H	β	FS
5 m	25	1.88
	30	1.56
	35	1.35
	40	1.20
	45	1.12
	50	1.04
	55	0.99
10 m	15	1.95
	20	1.47
	25	1.16
	30	0.96
20 m	12.5	1.66
	15.0	1.37
	17.5	1.16
	20.0	0.99
30 m	10	1.79
	12.5	1.41
	15.0	1.15
	17.5	0.97
40 m	10.0	1.61
	12.5	1.28
	15.0	1.05

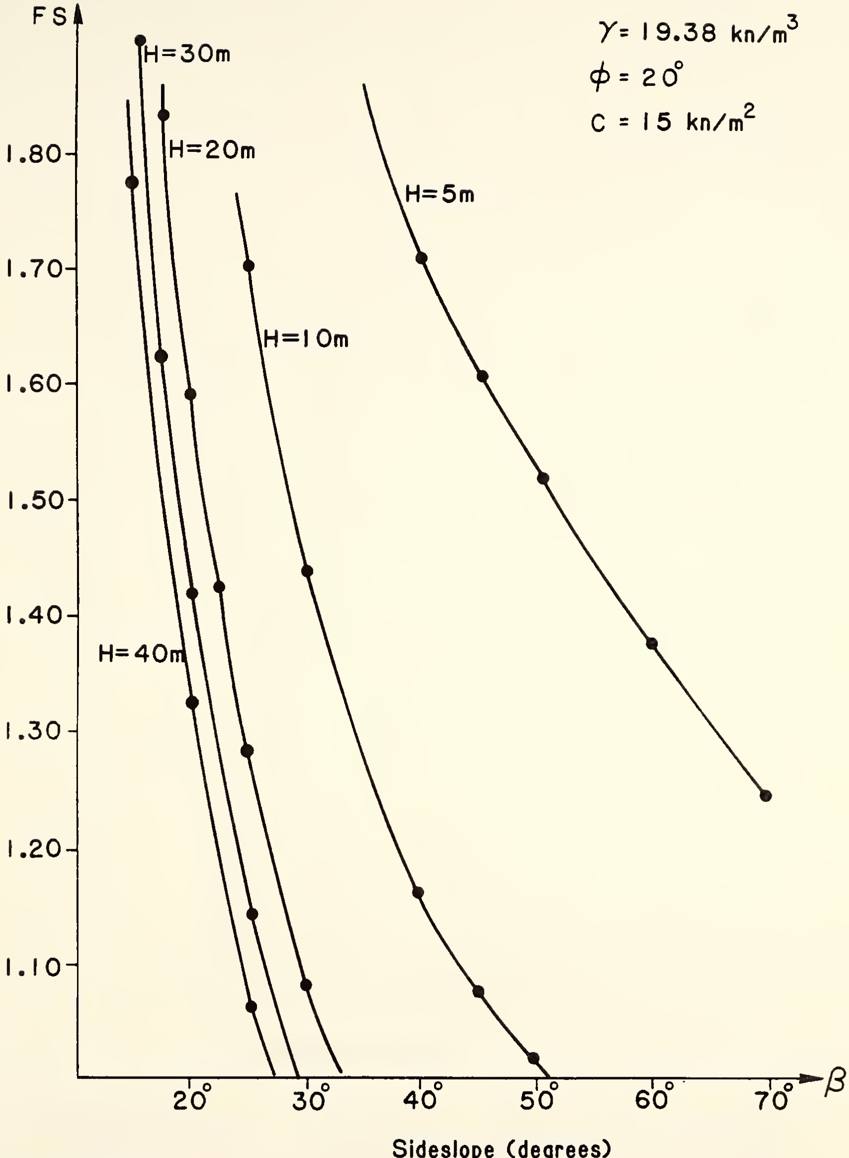


FIGURE 3.25

UNSATURATED LONGTERM INTRINSIC FACTOR OF SAFETY OF ST. CROIX CLAY EMBANKMENTS

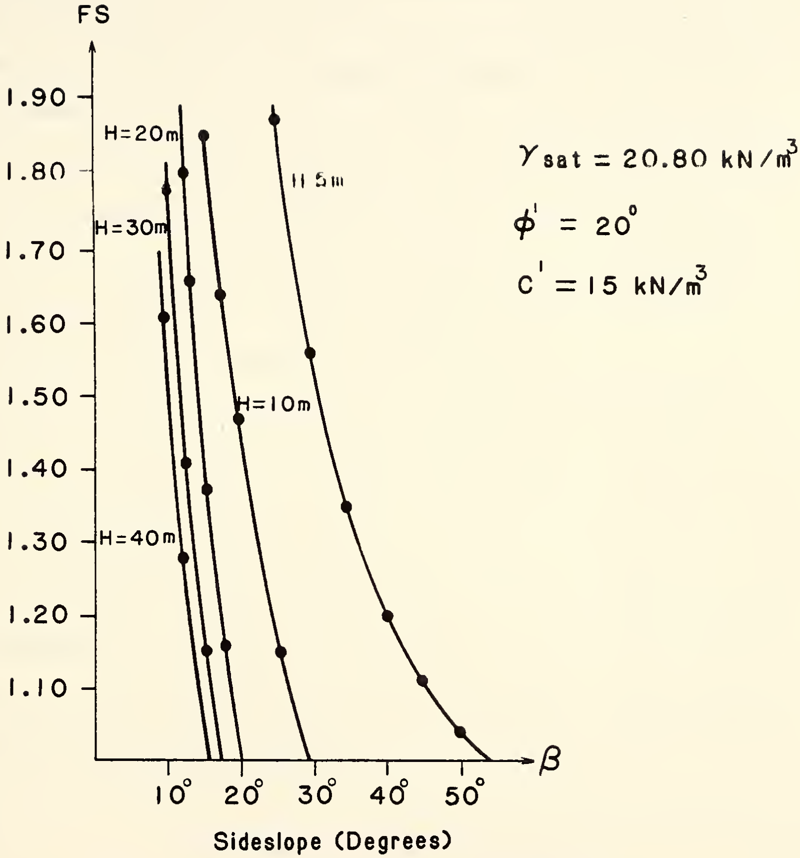


FIGURE 3.26 SATURATED LONGTERM INTRINSIC FACTOR OF SAFETY OF ST. CROIX CLAY EMBANKMENTS.

Interpretation of the Factor of Safety

This section addresses the engineering interpretation of a computed factor of safety and the choice of a minimum factor of safety to achieve embankment stability. The general rule of thumb in the past has been that a value of the factor of safety for earthworks between 1.3 and 1.5 is acceptable (Terzaghi and Peck, 1967). The lower limit is for maximum loading conditions and the upper limit is for service conditions (Meyorhoff, 1970). If the engineer designs an embankment slope with conventional methods of analysis and these values of the factor of safety, it is expected that the embankment will perform satisfactorily. Up to this point, the factor of safety that has been discussed has been a strength factor of safety. Since factors such as the dependence of the position of the critical surface on the factor of safety make the interpretation of the strength factor of safety difficult, supplemental methods of evaluating the proximity of a slope to limit equilibrium are useful. Two such methods are the geometric interpretation and the probabilistic approach.

The Geometric Interpretation

The factor of safety is usually defined as the ratio of the strength in the limit equilibrium state to the actual stress. The choice of strength as the variable for this comparison is arbitrary, however. It is equally valid to

define the factor of safety in terms of some relevant dimension of the slope geometry. Consider a typical slope in Figure 3.27a. Generally, the material parameters of the soil, ϕ, c, and γ are known. The height, H, and the width, W, are specified by need. The only variable that is not fixed is the sideslope, β. It seems appropriate, therefore, to define the factor of safety as the ratio of the sideslope at limit equilibrium, β_{cr}, to the actual sideslope, β, or, i.e.,:

$$FS_{\beta} = \beta_{cr}/\beta \qquad (3.11)$$

The value of the side slope at limit equilibrium, β_{cr}, may be obtained graphically by plotting the strength factor of safety vs. the side slope for fixed values of ϕ, c, γ and H. β_{cr} is the point on the curve that corresponds to a strength factor of safety of 1.0 (Figure 3.27b).

Example 3.3

It is desired to determine the relationship between the intrinsic strength factor of safety of an embankment and the β_{cr}/β ratio. The height of the embankment is 15.24 m (50 ft). The soil density, friction angle, and cohesion intercept are taken to be 19.34 kN/m^3 (123 pcf), 21°, and 14.85 kPa (310 psf), respectively, to simulate long-term behavior.

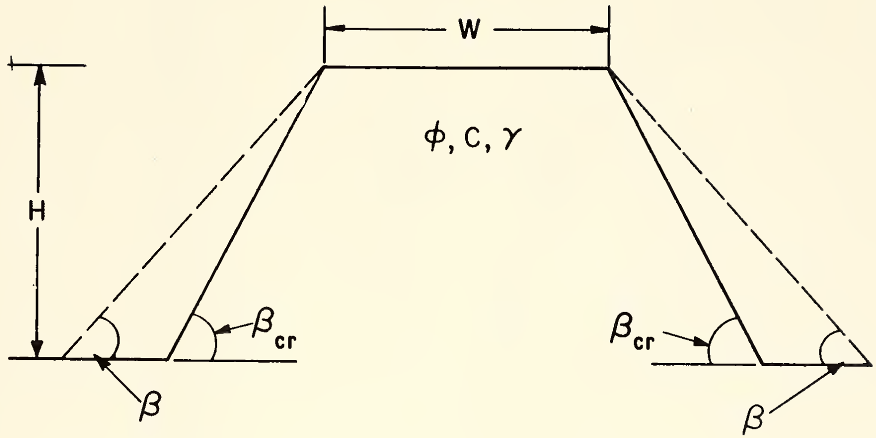


FIGURE 3.27 a

EARTHEN SLOPE CONSIDERED FOR DEFINING
THE SIDE SLOPE FACTOR OF SAFETY

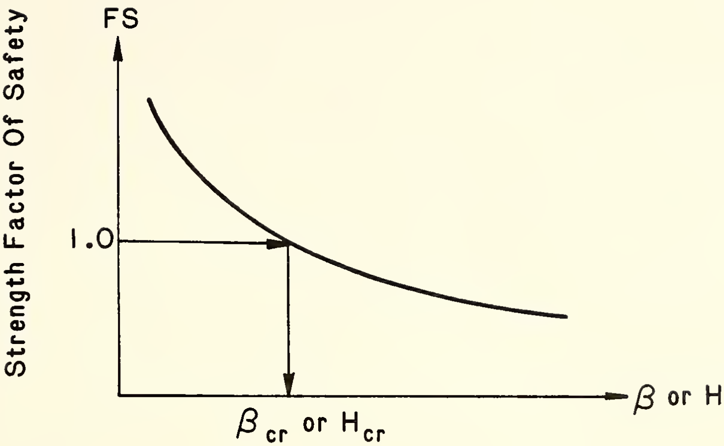


FIGURE 3.27 b

PROCEDURE TO DETERMINE THE LIMIT EQUILIBRIUM VALUES OF THE SIDESLOPE AND HEIGHT

The first step is to calculate the values of the intrinsic strength factor of safety for various side slopes. This can be done with the computer program in Appendix C. The results are given in Table 3.7 and Figure 3.28. The value of β_{cr} obtained from Figure 3.28 is 39.1^{o}. The ratio, β_{cr}/β, is plotted vs. the strength factor of safety in Figure 3.29. For this case, the ratio β_{cr}/β is always greater than the value of the strength factor of safety. One shortcoming of this approach occurs when there is no value of sideslope for which the factor of safety reduces to 1.0. In such a case the factor of safety can not be defined as β_{cr}/β. Fortunately, in such cases, the shear strength is so high that a stability analysis usually is not necessary.

A factor of safety based on the ratio of the height at limit equilibrium to the actual height, H_{cr}/H, may be defined in a manner analogous to the development of the side slope factor of safety by holding ϕ, c, γ, and β constant. The actual geometry of the embankment and the hypothetical geometry at limit equilibrium are shown in Figure 3.30. The graphical method for finding the critical height is illustrated in Figure 3.28.

<u>Example 3.4</u>

It is desired to find the variation of the ratio H_{cr}/H of an embankment vs. the intrinsic strength factor of safety. The side slope of the embankment is 1 to 2

Table 3.7 Sideslope Factor of Safety Calculation - Example 3.3

β^{o}	FS	β_{cr}/β
22	1.670	1.777
24	1.535	1.629
26	1.425	1.504
28	1.331	1.396
30	1.251	1.303
32	1.181	1.222
34	1.120	1.150
36	1.069	1.086
38	1.024	1.029
39	1.002	1.003
40	.984	.978

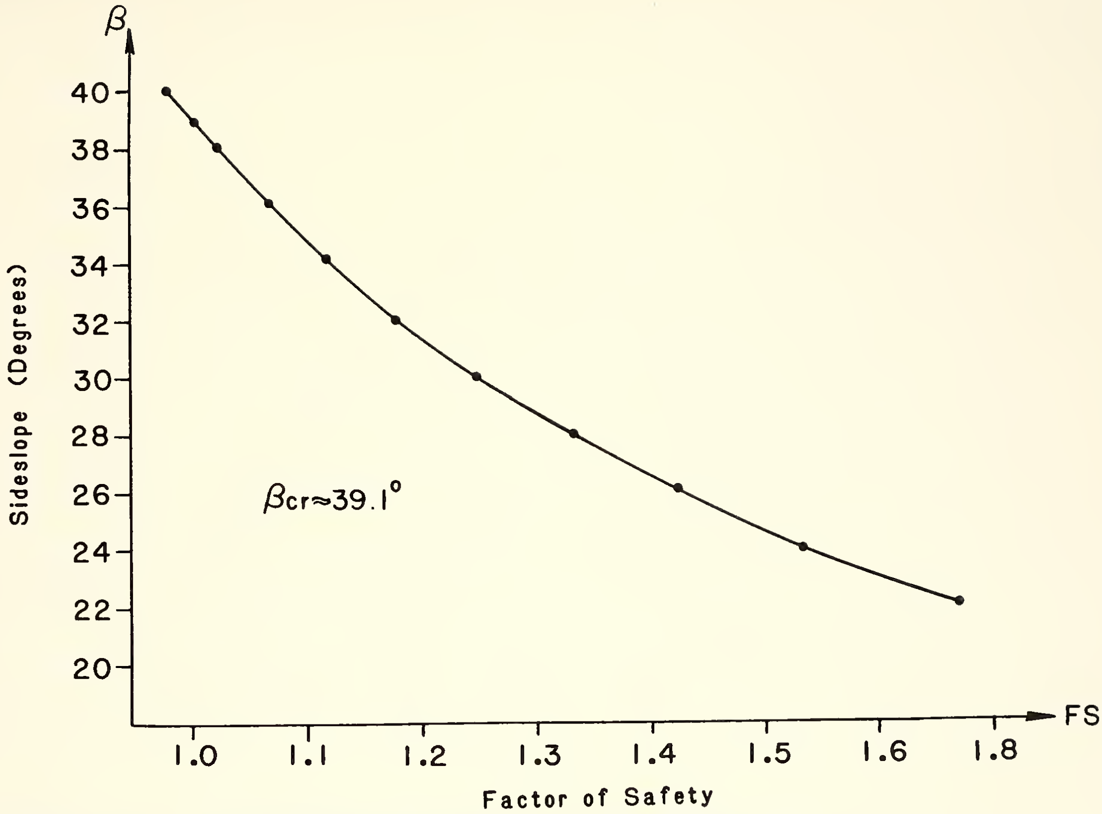


FIGURE 3.28 DETERMINATION OF β_{cr} – EXAMPLE 3.3

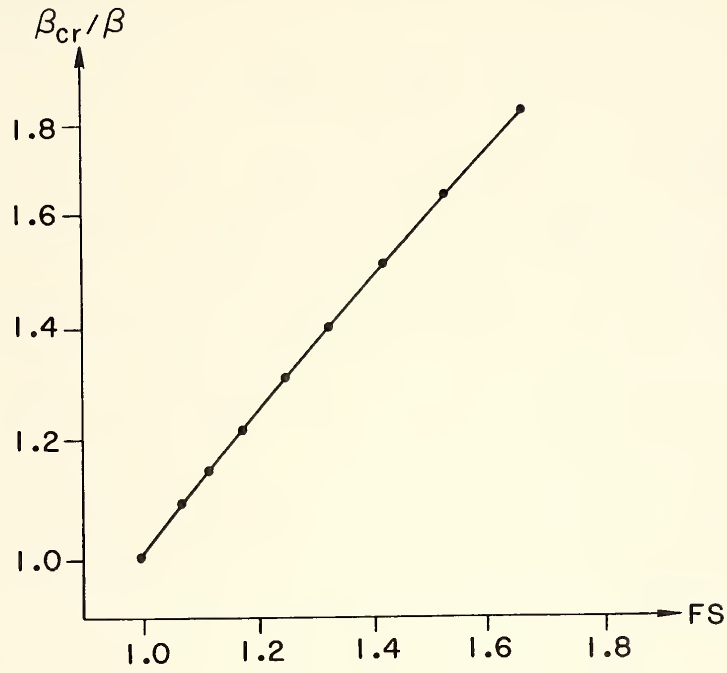


FIGURE 3.29 SIDESLOPE FACTOR OF SAFETY VS. STRENGTH FACTOR OF SAFETY – EXAMPLE 3.3

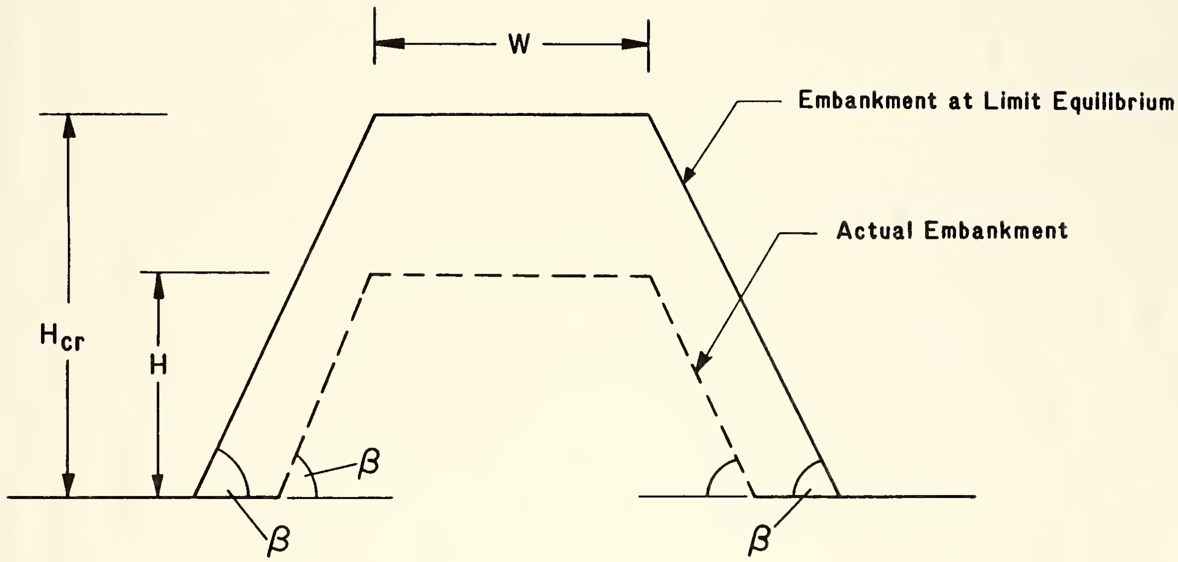


FIGURE 3.30 EARTHEN SLOPE CONSIDERED TO DEFINE THE FACTOR OF SAFETY BASED ON THE HEIGHT CRITERION

(26.57°). Assume the values of the soils material parameters are the same as in example 3.3. The height is variable. The strength factor of safety was obtained for various heights with the computer program in Appendix C. The results are given in Table 3.8 and Figure 3.31. The critical height is approximately 44.2 m (145 ft). The critical value is divided by each height corresponding to a value of the strength factor of safety. The results are given in Table 3.8 and Figure 3.32. The ratio, H_{cr}/H, is substantially larger than the value of the strength factor of safety. This is not always the case. For example, if γ, ϕ, and c were taken to be 19.59 kN/m^3 (124.6 pcf), 7.3°, and 95.02 kPa (1984 psf), respectively, to simulate the unsaturated short term strength of St. Croix clay compacted 2% wet of OMC, the results would be those shown in Figure 3.33. In this case the difference between the ratio H_{cr}/H and the intrinsic strength factor of safety is smaller. This example demonstrates that the strength factor of safety corresponds to different values of the H_{cr}/H ratio, depending on the values of ϕ and c. An analagous remark applies to the side slope ratio.

The Probabilistic Approach

The conventional definition of the factor of safety overlooks the variability of material parameters because it uses deterministic (single-valued) soil properties. The

Table 3.8 Factor of Safety Calculated Based on Height Criterion - Example 3.4

H (ft)	H_{cr}/H	FS
10	14.500	3.456
20	7.250	2.192
25	5.800	1.932
30	4.833	1.759
35	4.143	1.63
40	3.625	1.533
45	3.222	1.458
50	2.90	1.398
55	2.636	1.347
60	2.417	1.304
65	2.231	1.268
70	2.071	1.235
75	1.933	1.207
80	1.813	1.183
85	1.706	1.166
90	1.611	1.143
95	1.526	1.125
100	1.450	1.113
105	1.380	1.099
110	1.318	1.086
115	1.261	1.070
120	1.208	1.060
125	1.160	1.050
130	1.115	1.035
135	1.074	1.025
140	1.036	1.010

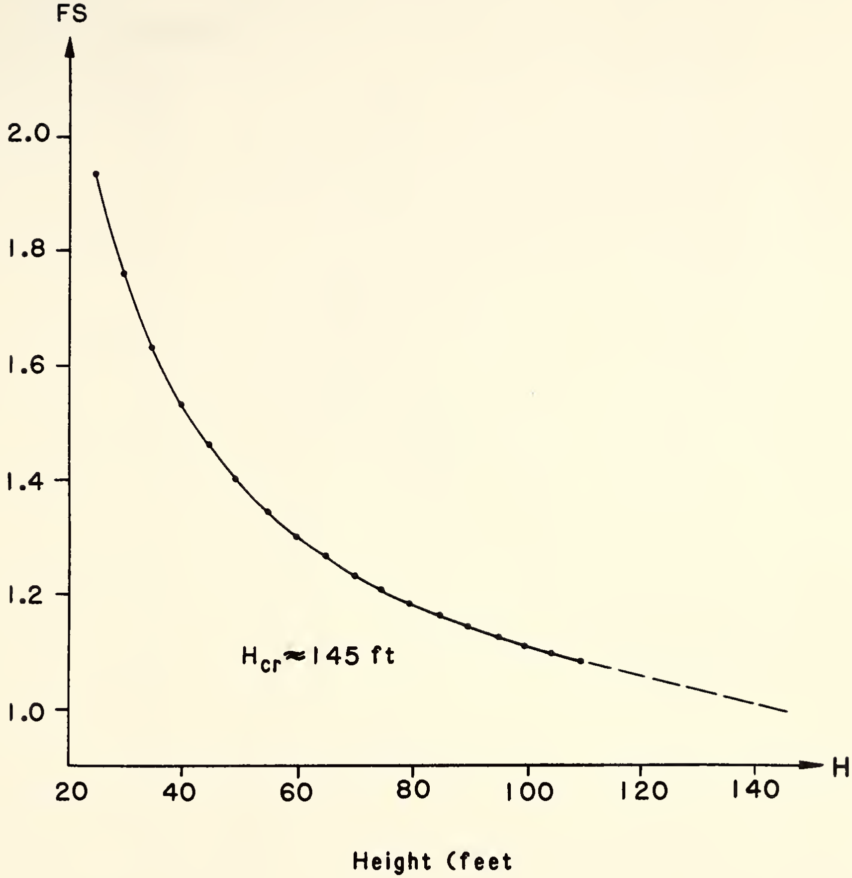


FIGURE 3.31 DETERMINATION OF H_{cr} – EXAMPLE 3.4

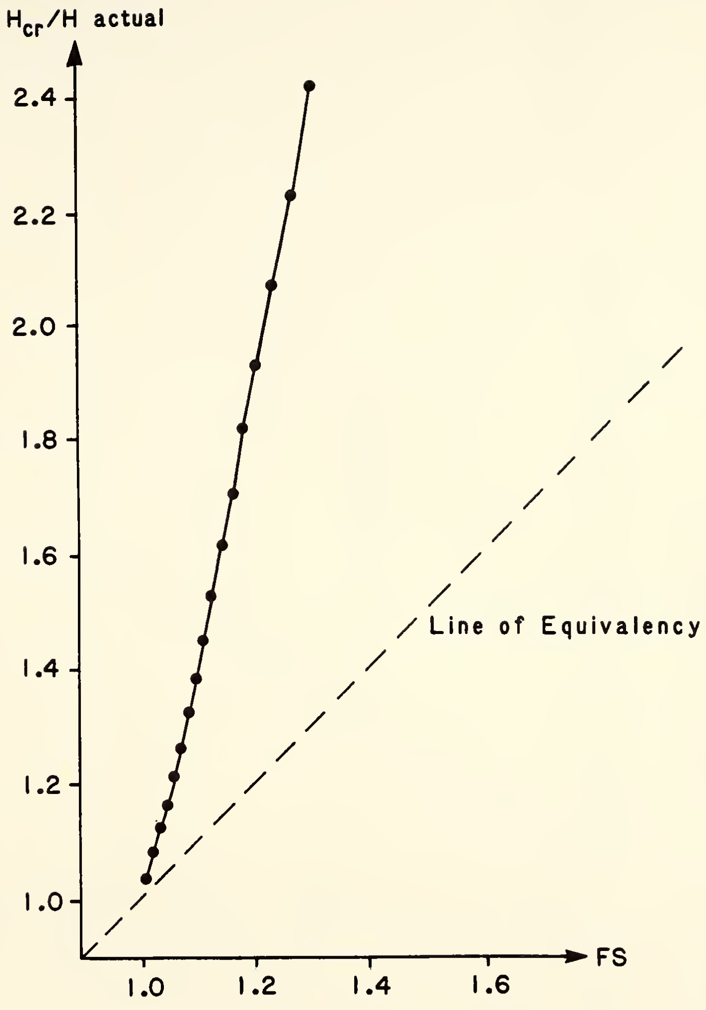


FIGURE 3.32 FACTOR OF SAFETY BASED ON THE HEIGHT CRITERION VS. THE STRENGTH FACTOR OF SAFETY—EXAMPLE 3.4

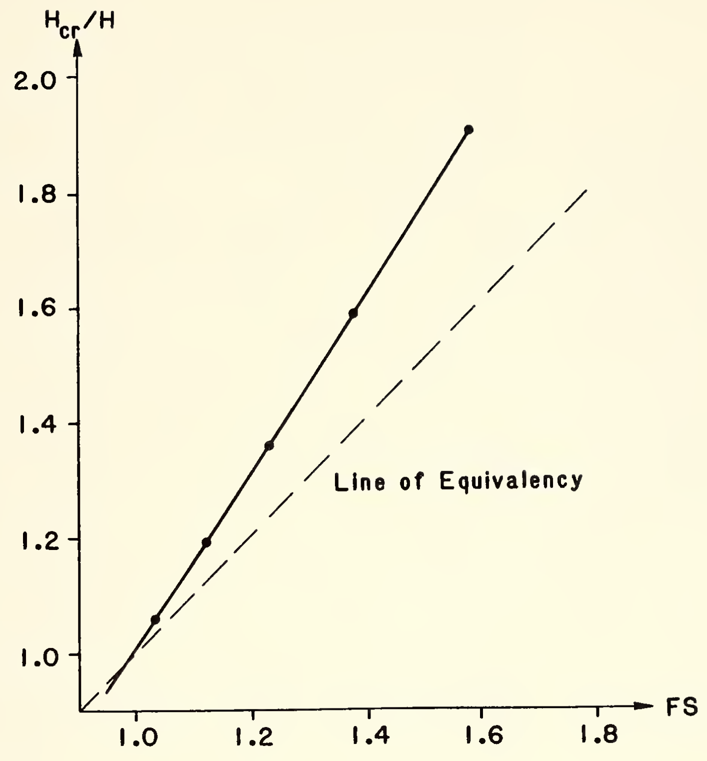


FIGURE 3.33 EXERCISE 3.4 REPEATED WET OF OPTIMUM

variability is only taken into account through the personal judgment exercised in the selection of the soil properties. Variability in soil properties arises through:

(1) material and sample non-homogeneities that do not represent the whole soil ("true" variability)

(2) sampling errors caused by disturbance during the sampling process

(3) errors that occur when tests are not performed according to a standard.

Uncertainty, however, is not limited to the variability observed in the basic variables. Analytical models and laboratory and field experiments are often only an idealized representation of reality. Predictions made on the basis of these models and experiments may be inaccurate and contain uncertainty. Therefore, the capacity of a slope to resist loading will not have a unique value. Similarly, the load (or demand) on the trial surface will have a distribution of values. These distributions can be represented by probability density functions. The probability that a slope will reach a state of limit equilibrium equals the probability that the capacity will be less than the demand (Yao, 1982), i.e.,

$$P_f = p(R<S) = \int_{R_{min}}^{S_{max}} f_S(s) \cdot F_R(s)\, ds \qquad (3.12)$$

where

R denotes strength (or capacity)

S denotes load (or demand)

f_S is the probability density function (PDF) of the load

F_R = cumulative distribution function (CDF) of the resistance.

R_{min} = minimum value of the strength (see Figure 3.34)

S_{max} = maximum value of the load (see Figure 3.34)

Capacity-demand problems are simplified in the case of slope stability analysis because the capacity-demand ratio for a slope of specified geometry and material parameters is identical to the factor of safety. Therefore, the probability of failure, P_f, is also equal to the probability that the factor of safety is less than one, or

$$P_f = P(FS < 1.0) \tag{3.13}$$

This calculation is performed by integrating the probability density function of the factor of safety up to a value of one. The density function of the factor of safety depends on the distributions of the soil density and shear strength variables. When the distribution of these variables is known, the density function of the factor of safety may be obtained by Simulation (Yao, 1982).

Simulation is essentially a controlled statistical sampling technique which can be used to study complex

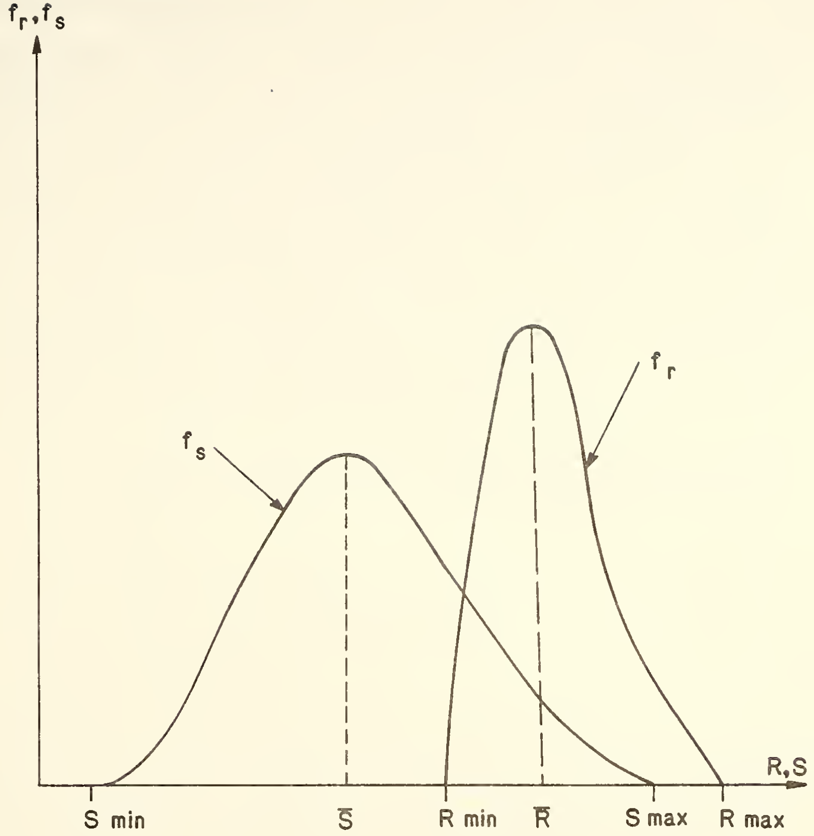


FIGURE 3.34 PROBABILITY DENSITY (OR DISTRIBUTION) FUNCTIONS OF CAPACITY AND DEMAND

stochastic systems when analytical and/or numerical techniques do not suffice. A necessary part of any such procedure is an algorithm for random number generation. A random number generator produces sequences with probability density functions that are uniformly distributed between 0 and 1 and that possess the appearance of randomness. Most computer systems have a built-in random number generator. The inverse transformation method, often called the Monte-Carlo method of simulation, is used to generate non-uniform random number X, with cumulative distribution function $F_x(x)$. The algorithm is very simple:

1. Generate number Q uniformly distributed between 0 and 1.
2. Return $X = F_x^{-1}(Q)$
   (F_x^{-1} is the inverse function corresponding to F_x)

This algorithm assumes that the equation $F_x(x) = Q$ can be solved explicitly. Other distributions can be simulated by direct generation methods such as composition methods and rejection-acceptance methods (Fishman, 1978). The probability of failure equals the cumulative distribution function of the factor of safety up to a value of one, i.e.,

$$P_f = F_{FS}\ (FS = 1.0) \tag{3.14}$$

Since the Monte-Carlo method uses the actual distributions

of the variables that affect the factor of safety, it can, in principle, generate the actual density function of the factor of safety. Unfortunately, this requires a very large number of simulations and consequently, a great amount of computational effort.

A simpler approach used in practice to obtain the probability of failure is to assume a distribution of the factor of safety and to generate statistical moments of this distribution from the statistical moments of the dependent variables (density and shear strength parameters). Two procedures to obtain these moments are the Taylor Series expansion method and the Point Estimates Method (Yao, 1982).

The Taylor Series expansion requires partial derivatives of the factor of safety with respect to each of the variables that affect the factor of safety (Harr, 1977). The difficulty of evaluating these derivatives limits the usefulness of the Taylor Series expansion for modelling the density function of the factor of safety.

The Point-Estimates Method is an approximate procedure for calculating the statistical moments of the factor of safety which does not require evaluation of the derivatives of the factor of safety relative to the variables affecting it (Rosenblueth, 1975). This method approximates the statistical moments of a function Y (i.e., the factor of safety) by "replacing" the distribution of the variables

affecting the function with point estimates (or weights) at properly selected values of the variables. For example, the estimated value of the n^{th} statistical moment of the function Y that depends on two random variables may be found with the following two-point procedure:

$$E(Y^n) = P_{++}y_{++} + P_{+-}y^n_{+-} + P_{-+}y^n_{-+} + P_{--}y^n_{--} \qquad (3.15)$$

$$y_{\pm\,\pm} = Y[\bar{X}_1 \pm \sigma_{X1}, \bar{X}_2 \pm \sigma_{X2}] \qquad (3.16)$$

and

P = weighting factor

X_1, X_2 = random variables

σ_{X1}, σ_{X2} = standard deviations of the random variables

$\bar{X}$ = mean value of the variable, X

If X_1 and X_2 have symmetric distributions, the weighting factor is 1/4. The point estimates method can be adapted to handle any number of correlated and/or uncorrelated random variables (Rosenblueth, 1975).

To illustrate the application of the probabilistic approach to slope stability problems, the friction-circle slope stability program (Appendix C) was adapted to calculate the probability of failure of a simple slope. The resulting program is given in Appendix E. This program assumes that the material parameters defining the soil strength and density have symmetric distributions. These

distributions are defined with mean values, coefficients of variation, and upper and lower bounds. The material parameters are assumed to be uncorrelated. The probability density function of the factor of safety is assumed to be beta distributed (Appendix D). The mean and variance (i.e., the first two statistical moments) of the factor of safety are obtained with a two-point point estimates procedure. The lower bound of the distribution of the factor of safety is the factor of safety that is obtained using the maximum value of the soil density and the minimum values of the Mohr-Coulomb parameters, ϕ and c. The upper bound is the factor of safety that is obtained using the minimum value of the density and maximum values of the Mohr-Coulomb parameters. Finally, the probability of failure is calculated by integrating equation 3.13 numerically.

Example 3.5

It is desired to determine the probability of failure of the slope shown in Figure 3.35. The average ratio $\bar{\gamma}H/\bar{c}$ is 25/3

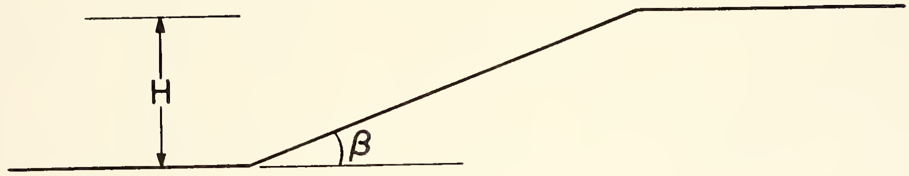
where

$\bar{\gamma}$ = the mean value of the soil density

$\bar{c}$ = the mean value of the cohesion intercept of the soil

H = the height of the slope

The variability of the material parameters is given in Table



$\bar{\phi} = 5^\circ$

$\bar{\gamma} = 20\ kN/m^3$

$\bar{c} = 100kPa$

$H = 41.66\ m$

$\beta = 20^\circ$

FIGURE 3.35 SLOPE — EXAMPLE 3.5

3.9. The mean factor of safety corresponding to the mean values of the soil density and the Mohr-Coulomb strength parameters is found to be 1.28 using the friction circle method. The program in Appendix E is used to quantify the effect of the variability of the cohesion intercept on the probability of failure. The results are reported in Table 3.10 and Figure 3.36. An increase in the variability of the cohesion intercept increases the probability of slope failure. Therefore, slopes with equal mean factors of safety are not necessarily equally safe. Historical records indicate that earth dams designed with ordinary techniques for a factor of safety of 1.3 to 1.5 have a rate of failure on the order of one in two thousand (Meyerhof, 1970). However, further work needs to be done in order to recommend design values of the probability of failure for various situations such as sudden drawdown and long-term conditions.

Frequently, the probability of failure that is calculated based on ordinary estimates of material variability is much higher than the values reported by Meyerhoff. This discrepancy arises because designers typically use a lower bound on their strength parameters rather than mean values when they calculate the factor of safety. If mean values of the strength parameters were used, a higher factor of safety has to be employed to assure adequate performance.

Table 3.9 Material Parameters - Example 3.5

Parameter	Value
$\bar{\phi}$	5°
ϕ_{min}	$0.6\ \bar{\phi}$
ϕ_{max}	$1.4\ \bar{\phi}$
γ_{min}	$.85\ \bar{\gamma}$
γ_{max}	$1.15\ \bar{\gamma}$
c_{min}	$.66\ \bar{c}$
c_{max}	$1.33\ \bar{c}$

Parameter	Coefficient of Variation
ϕ	0.2
γ	0.1
c	variable

Table 3.10 Probability of Failure vs. Variability of Cohesion - Example 3.5

β	FS	μ_c	P_f
20°	1.23	.01	.016
		.05	.023
		.10	.047
		.20	.145
		.30	.253
		.33	.284
30°	1.05	.01	.226
		.05	.292
		.10	.330
		.20	.401
		.30	.449
		.33	.462

β = sideslope
FS = factor of safety based on the mean value of ϕ, c, and γ.
μ_c = coefficient of variation of the cohesion
P_f = probability of failure.

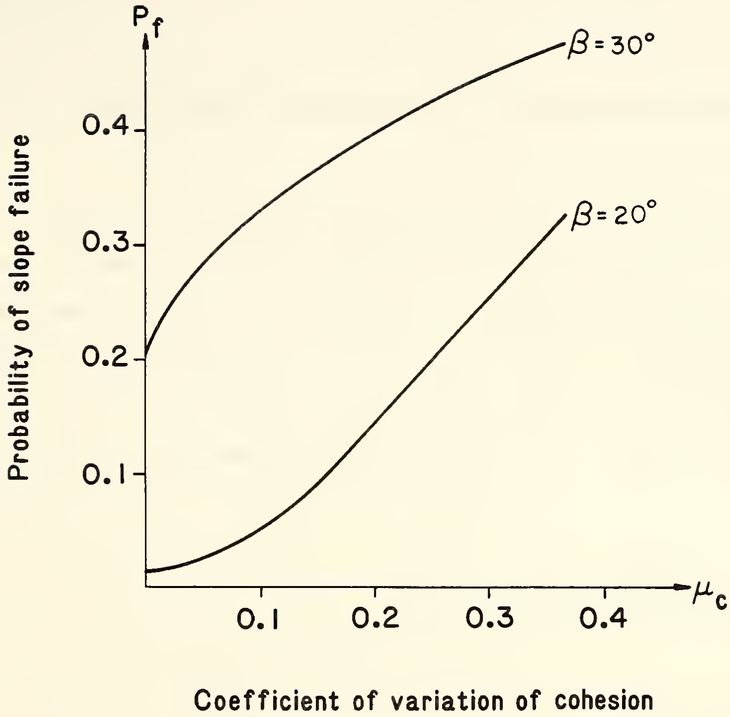


FIGURE 3.36 PROBABILITY OF FAILURE VS. COEFFICIENT OF VARIATION OF THE COHESION INTERCEPT – EXAMPLE 3.5

IV - SETTLEMENT CONSIDERATIONS FOR EMBANKMENT DESIGN

Once an embankment has been checked for intrinsic and overall stability of its sideslopes, the settlement of the embankment should be investigated. Embankment settlement is comprised of displacements that occur within the embankment itself as well as compression of fine grained soil layers that underlie the embankment. Displacements within the embankment itself are caused principally by:

1) partially saturated compression under the body forces of the fill. This occurs as rapidly as the fill is constructed (DiBernardo, 1979).
2) volume change due to an increase in moisture content of the compacted embankment. This is thought to be the major source of displacements in compacted clay embankments.

Compression of the fine grained soil layers below the embankment consists of:

1) immediate settlement that occurs at constant volume. Immediate settlement is completed at the end of

embankment construction (consequently, it has no effect on the embankment performance and may be neglected).

2) time dependent consolidation settlement that occurs as excess pore pressure induced by the embankment construction dissipates

3) secondary compression settlement that occurs after time dependent consolidation is complete (calculation of the secondary compression settlement is frequently omitted because it is assumed to be small compared to the consolidation settlement).

Saturation Induced Displacement of Compacted Clay Embankments

When a sample of compacted clay becomes saturated, it may either swell or settle depending on the mineralogy of the clay, the presaturation water content, the compactive effort, and the overburden pressure. Prediction models have been developed for the volume change due to saturation for laboratory compacted St. Croix clay (DiBernardo, 1979). An extension of this model for field compacted soils with a range of plasticity index values was proposed by Lin (1981). However, since the data base for these models is limited, it is recommended that the volume change due to saturation be estimated from tests on the soil that will be used in the embankment.

To insure that the testing will best simulate the volume change in the embankment due to saturation, the following guidelines should be followed:

1) The soil sample should be compacted to the expected state of compaction in the field according to the procedure described in Chapter II.
2) The presaturation water content of the soil should be the same as is expected in the actual embankment.
3) The soil sample should be subjected to a pressure equal to the overburden pressure in the embankment. Therefore, to simulate the variation of volume change with depth, the test samples must be subjected to a range of overburden pressures.
4) Measurement of volume change of the soil sample should be made from the time that the soil is back-pressure saturated until the volume change ceases. For details of the test procedure, see DiBernardo (1979).

If it is assumed that all volume change occurs vertically, it is possible to estimate the settlement of the surface of the embankment with the following expression:

$$S = \frac{1}{100} \int_0^H U(z)\, dz \qquad (4.1a)$$

where

S = the settlement due to saturation

$U(z)$ = the % volume change at the depth z

z = the depth beneath the top of the embankment

H = the height of the embankment

If the distribution $U(z)$ can be idealized as a sequence of strata each with its own uniform value of U, then equation 4.1a may be evaluated numerically with the following expression

$$S = \frac{1}{100} \sum_{i=1}^{n} U_i \cdot \Delta z_i \tag{4.1b}$$

where

U_i = the % volume change of stratum i

Δz_i = the thickness of stratum i

n = the number of strata

It is interesting to note that it is possible for the upper portion of an embankment to be swelling while the lower portion is settling. In fact, it is theoretically possible to build an embankment whose net saturation settlement is nill by specifying the compaction so that there are compensating zones of swelling and settling soil within the embankment. Generally, however, it is best to design the embankment to settle slightly because swelling of soil beneath the road bed can cause severe pavement distress.

Consolidation Settlement of Compressible Soil Layers Beneath the Embankment

Magnitude of Settlement

When a saturated clay sample is subjected to an axial stress change in a standard consolidation test, an excess pore pressure equal to the stress change is induced in the sample. As time proceeds, the excess pore pressure will dissipate and the sample will settle by a volume equal to the volume of the dissipated pore water. The relationship between the axial stress and the void ratio after all of the excess pore pressure has been dissipated is described in Figure 4.1a. For purposes of analysis, the relationship may be replaced with the representation shown in Figure 4.1b which is defined by the compression index, C_c, the recompression index, C_r, and the preconsolidation pressure, σ'_p.

If the existing axial pressure, σ'_v, is equal to the preconsolidation pressure, the soil is normally consolidated. The settlement of the soil due to the axial pressure change is:

$$S = \frac{H}{1 + e_o} C_c' \log \frac{\sigma'_v + \Delta\sigma}{\sigma'_p} \qquad (4.2)$$

where

S = the settlement of the clay layer

H = the thickness of the clay layer

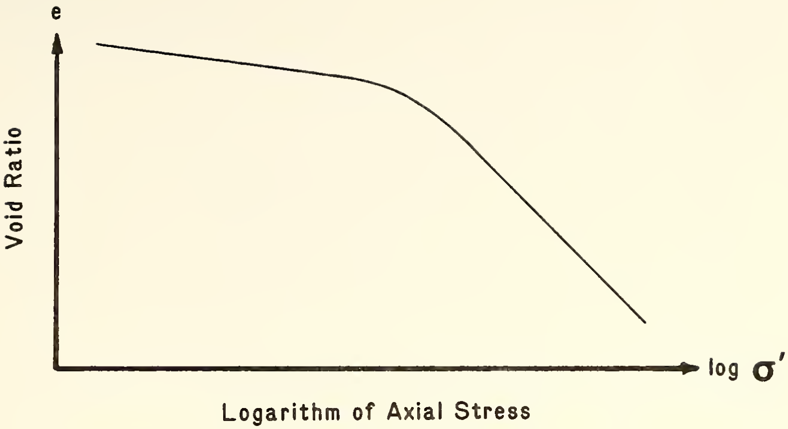


FIGURE 4.1a TYPICAL e–log σ' RELATIONSHIP

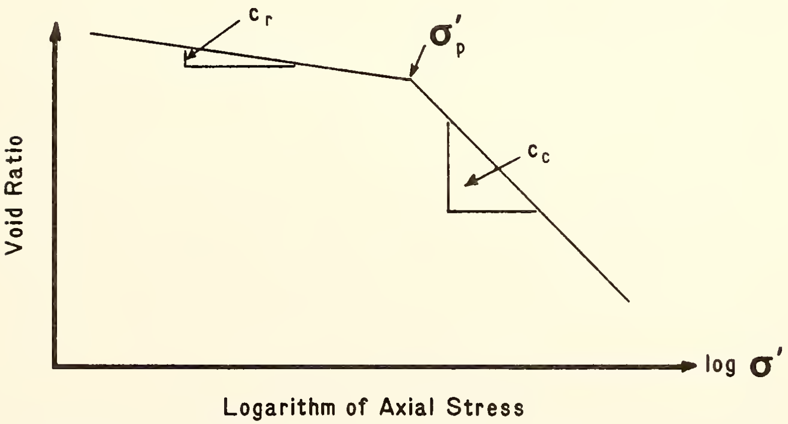


FIGURE 4.1b SIMPLIFICATION OF TYPICAL e–log σ' RELATIONSHIP FOR ANALYTICAL PURPOSES

$\Delta\sigma$ = the increase in axial stress of the clay layer

e_o = the initial void ratio of the clay layer

σ'_v = the existing overburden pressure

If the existing overburden pressure is less than the preconsolidation pressure, the soil is overconsolidated. The settlement due to the axial pressure change is:

$$S = \frac{H}{1 + e_o} [C_r \log \frac{\sigma'_v + \Delta\sigma}{\sigma'_v}] \qquad (4.3)$$

when $(\sigma'_v + \Delta\sigma) \leq \sigma'_p$, or

$$S = \frac{H}{1 + e_o} [C_r \log \frac{\sigma'_p}{\sigma'_v} + C_c \log \frac{\sigma'_v + \Delta\sigma - \sigma'_p}{\sigma'_p}] \qquad (4.4)$$

when $(\sigma'_v + \Delta\sigma) > \sigma'_p$.

If the existing overburden pressure is greater than σ'_p, the soil is underconsolidated. The settlement due to the axial pressure change is:

$$S = \frac{H}{1 + e_o} C_c \log \frac{\sigma'_v + \Delta\sigma}{\sigma'_p} \qquad (4.5)$$

These expressions for settlement are exact provided that

1) the values of e_o, C_r, C_c, σ'_v, and σ'_p are the same in the consolidating layer as measured in

the consolidation test

2) the stress change in the consolidating layer does not vary with depth

3) the excess pore pressure caused by the embankment loading is the same as in the consolidation test.

Deviations from the first two assumptions may be accounted for by dividing the consolidating layer into artificial strata. Deviations from the third assumption may be accounted for by use of a correction factor that will be discussed later.

To determine the value of e_o in each stratum, one can assume that the difference in the initial void ratio between the center of the stratum and the position of the soil sample, Δe_o, will be equal to the change in void ratio that would occur if the soil sample were to move along the e - log σ′ curve by a stress change equal to the difference in overburden pressure between the position of the soil sample and the center of the stratum.

If the center of the stratum is above the elevation of the sample, Δe_o will be positive. If, in addition, the soil is overconsolidated above the elevation of the sample

$$\Delta e_o = C_r \log (\sigma_{sample}/\sigma'_{vo}) \qquad (4.6a)$$

where

σ_{sample} = the overburden pressure in the soil sample

σ'_{vo} = the overburden pressure in the stratum

If the soil above the elevation of the sample is considered to be underconsolidated:

$$\Delta e_o = 0 \tag{4.6b}$$

If the soil is underconsolidated at pressures above the preconsolidation pressure of the sample, σ'_p, and overconsolidated at pressures below σ'_p

$$\Delta e_o = C_r \cdot \log (\sigma'_p/\sigma'_{vo}) \tag{4.6c}$$

If the soil is normally consolidated at pressures above σ'_p and overconsolidated at pressures beneath σ'_p

$$\Delta e_o = C_c \cdot \log (\sigma'_{sample}/\sigma'_p) + C_r \cdot \log (\sigma'_p/\sigma'_{vo}) \tag{4.6d}$$

If the center of the stratum is below the elevation of the sample, Δe_o will be negative. If the soil is overconsolidated beneath the elevation of the sample

$$\Delta e_o = - C_r \cdot \log (\sigma'_{vo}/\sigma'_{sample}) \tag{4.6e}$$

If the soil is overconsolidated at pressures below σ'_p and normally consolidated at pressures above σ'_p,

$$\Delta e_o = - C_r' \log (\sigma'_p/\sigma_{sample}) - C_c' \log (\sigma'_{vo}/\sigma'_p) \qquad (4.6f)$$

If the soil is overconsolidated throughout its stress path:

$$\Delta e_o = - C_r' \log (\sigma'_p/\sigma_{sample}) \qquad (4.6g)$$

If the soil is underconsolidated throughout its stress path:

$$\Delta e_o = 0 \qquad (4.6h)$$

Like the void ratio, σ'_p varies with depth in a clay layer. Typically, σ'_p decreases with depth until the soil is normally consolidated. Thereafter it assumes a value equal to the overburden pressure (Holtz and Kovacs, 1981). This is illustrated in Figure 4.2a. The variation of σ'_p with depth may be represented by two straight-line segments (Figure 4.2b). Once the values of σ'_p at the endpoints of the two segments are specified, the value of σ'_p may be obtained by interpolation in any stratum. Developing the σ'_p profile requires more consolidation tests than are normally run. This is not prohibitive, however, because these consolidation tests can be run as constant gradient tests and completed in one work day.

The change in vertical pressure due to the embankment load can be approximated with the expressions presented in Appendix F. C_c and C_r are assumed constant for a given clay layer.

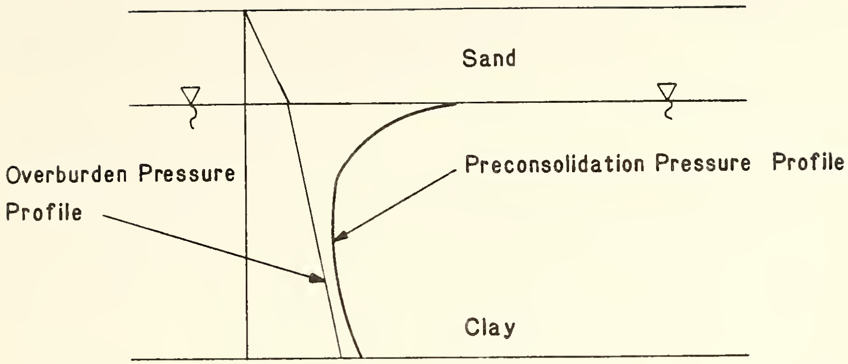


FIGURE 4.2a

TYPICAL PRECONSOLIDATION PRESSURE PROFILE

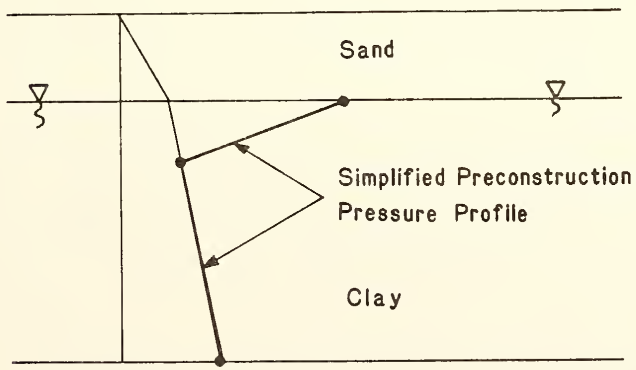


FIGURE 4.2b

SIMPLIFIED PRECONSOLIDATION PRESSURE PROFILE

Once the values of e_o, C_r, C_c, σ'_v, σ'_p and $\Delta\sigma$ are determined in each stratum, the settlement of the consolidating layer may be calculated by evaluating equations 4.2 - 4.5 for each stratum. The computer program in Appendix G has been provided to facilitate computations.

<u>Example 4.1</u>

It is desired to make a preliminary estimate of the consolidation settlement of the embankment built over the clay layer shown in Figure 4.3 without performing consolidation tests. The compression index may be estimated using an appropriate correlation with standard index tests (Terzaghi and Peck, 1967). The recompression index is assumed to be one tenth of the compression index. For purposes of this example, σ'_p was taken to be 95.76 kPa (2000 psf) at the depth of the sample. σ'_p was assumed to equal the overburden pressure when the overburden pressure exceeds 95.76 kPa.

It is instructive to consider the stress changes in the clay layer caused by the embankment before actually calculating settlement. This was done with the computer program in Appendix F. The results, which are shown in Table 4.1, indicate that:

1) The increase in vertical stress at a given depth in the clay layer is approximately constant across the embankment
2) The vertical stress increase does not attenuate

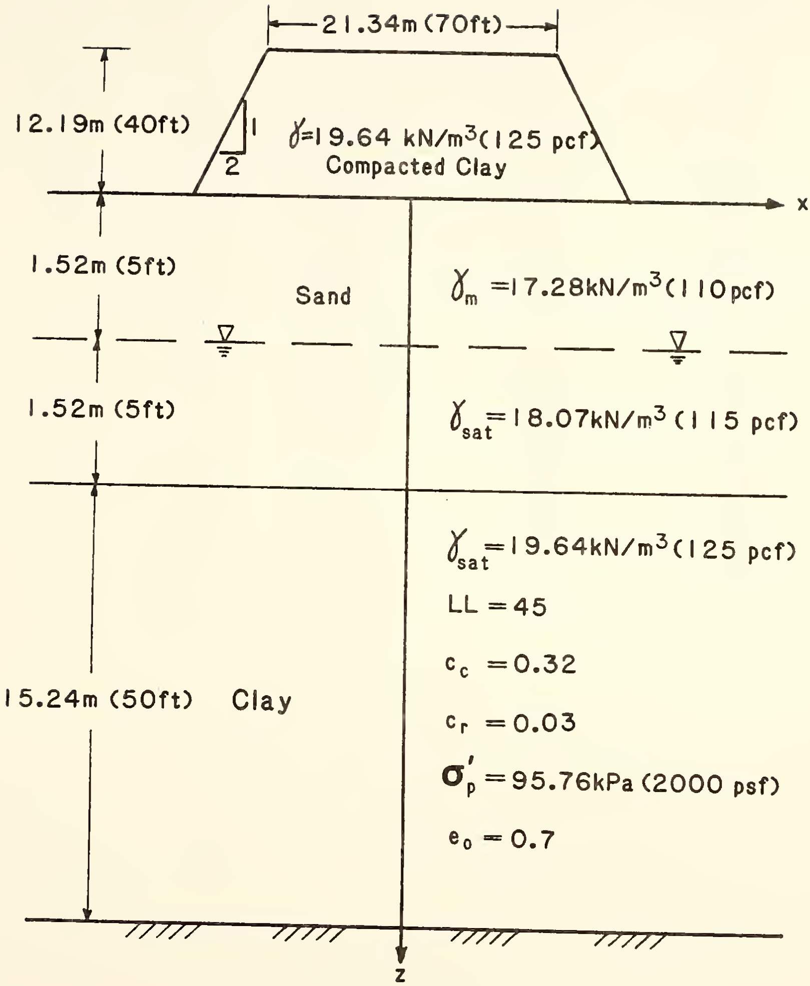


FIGURE 4.3

EMBANKMENT — EXAMPLE 4.1

Table 4.1 Stress Change Beneath Embankment - Figure 4.3

x(m)	z(m)	$\Delta\sigma_z$(kPa)	$\Delta\sigma_x$(kPa)	$\Delta\tau_{xz}$(kPa)
0.00	3.05	239.0	173.7	-74.4
0.00	4.57	238.0	163.4	-72.3
0.00	6.10	236.3	153.8	-69.6
0.00	7.62	233.8	69.0	-66.6
0.00	9.14	230.7	61.2	-63.2
0.00	10.67	228.5	54.3	-59.9
0.00	12.19	222.9	48.3	-56.5
0.00	13.72	218.3	42.9	-53.2
0.00	15.24	213.5	38.2	-50.0
0.00	16.76	208.6	34.0	-47.0
0.00	18.29	203.6	30.4	-44.1
10.67	3.05	229.9	148.8	8.0
10.67	4.57	225.1	118.5	8.0
10.67	6.10	220.3	94.4	6.5
10.67	7.62	215.6	75.4	4.5
10.67	9.14	210.9	60.6	2.3
10.67	10.67	206.1	49.1	0.5
10.67	12.19	201.4	40.1	-1.0
10.67	13.72	196.8	33.0	-2.1
10.67	15.24	192.2	27.4	-2.9
10.67	16.76	187.7	22.9	-3.5
10.67	18.29	183.2	19.3	-3.9

appreciably with depth.

3) The increase in horizontal stress is small compared to the increase in vertical stress everywhere except near the top of the clay layer.

4) Although the clay layer is moderately overconsolidated, the vertical stress increase occurs principly at values higher than the preconsolidation pressure.

Since most of the parameters affecting the consolidation vary with depth in the clay layer, the layer must be divided into a sufficient number of strata to insure adequate accuracy. This can be achieved with the computer program in Appendix G. The settlement at the centerline of the embankment shown in Figure 4.3 is given in Table 4.2 as a function of the number of strata, n. Five or more strata will suffice in this case. The settlement at the embankment centerline is 1.3 m. Once the number of strata to be used in the analysis is known, the lateral variation of the settlement across the embankment can be calculated. The results are shown in Table 4.3.

As noted before, the difference in the stress paths of the consolidation test and the clay layer beneath the embankment will give rise to the generation of different amounts of excess pore pressure, and consequently to different amounts of settlement. An approximate method for

Table 4.2 Settlement vs. Number of Strata - Example 4.1

n	S(m)
1	1.372
2	1.305
3	1.295
4	1.308
5	1.295
6	1.298

Table 4.3 Settlement and Differential Settlement Along Profile of Embankment - Example 4.1

x(m)	S(m)
0.000	1.298
1.524	1.298
3.048	1.292
4.572	1.289
6.096	1.280
7.620	1.268
9.144	1.250
10.668	1.228

dealing with this discrepancy is discussed in the following paragraphs.

The settlement of a compressible layer can be defined as:

$$S_{field} = \int_0^H a_v \cdot u \cdot dz \qquad (4.8)$$

where

a_v = the coefficient of vertical compressibility of the soil

u = the excess pore pressure due to loading

H = the thickness of the soil layer

dz = the thickness increment in the soil layer

In the field the excess pressure will be (Holtz and Kovacs, 1981):

$$u = B[\Delta\sigma_{oct} + a \cdot \Delta\tau_{oct}] \qquad (4.9)$$

where

$\sigma_{oct} = (\sigma_1 + \sigma_2 + \sigma_3)/3$

$\tau_{oct} = \frac{1}{3}\, [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$

$\sigma_1, \sigma_2, \sigma_3$ = principal stresses

a, B = pore pressure parameters that are determined experimentally.

The excess pore pressure generated in a consolidation test is equal to the change in vertical stress, i.e.,

$$u = \Delta\sigma_v \tag{4.10}$$

It follows that the value of the settlement in the field will be

$$S_{field} = \int_0^H a_v \cdot B\, [\Delta\sigma_{oct} + a \cdot \Delta\tau_{oct}]\, dz \tag{4.11}$$

and that the value of settlement if excess pore pressures are equal to those in a consolidation test will be

$$S_{lab} = \int_0^H a_v \cdot \Delta\sigma_v \cdot dz \tag{4.12}$$

By taking the ratio of the settlements in equations 4.11 and 4.12, a correction factor to apply to settlements computed with the laboratory pore pressures may be obtained (Skempton and Bjerrum, 1957). This correction factor is:

$$\mu = \frac{S_{field}}{S_{lab}} = \frac{\int_0^H a_v \cdot B \cdot [\Delta\sigma_{oct} + a \cdot \Delta\tau_{oct}]\, dz}{\int_0^H a_v \cdot \Delta\sigma_v \cdot dz} \tag{4.13}$$

Ordinarily, the parameter, B, is 1.0 for saturated clays. The parameter, a, depends on the stress path. The results of Example 4.1 show that the stress path beneath the

embankment is essentially triaxial compression. For this case, the parameter, a, may be obtained with the following expression

$$a = 3(A_{ac} - \tfrac{1}{3})/\sqrt{2} \tag{4.14}$$

where $A_{ac} = \Delta u/(\Delta\sigma_1 - \Delta\sigma_3)$, is measured in a triaxial test. The parameter, a, depends on the amount of strain. When the soil is treated as an isotropic elastic solid (which incidentally, is the assumption that is made in calculating the stress changes caused by the embankment) the parameter, a, is zero.

Assuming that the stress-strain behavior of a soil may be idealized as in Figure 4.1b, the value of the coefficient of axial compressibility in the normally consolidated range may be taken to be

$$a_v = \frac{C_c/\ln 10}{(1 + e)\,\sigma'_v} \tag{4.15}$$

Equation 4.13 may be simplified by assuming that a_v is constant and that B equals unity. The resulting expression is

$$\mu = \frac{\int_0^H (\Delta\sigma_{oct} + a\cdot\Delta\tau_{oct})\,dz}{\int_0^H \Delta\sigma_v\cdot dz} \tag{4.16}$$

Assuming that the soil may be considered to be an isotropic elastic solid, this may be further simplified to

$$\mu = \frac{\frac{1}{3}\int_0^H (\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3)\cdot dz}{\int_0^H \Delta\sigma_v \cdot dz} \qquad (4.17)$$

<u>Example 4.2</u>

It is desired to calculate the settlement correction factor for the centerline of the embankment in example 4.1. The values of the stress changes due to the embankment load (obtained with the computer program in Appendix F) are presented in Table 4.4. Using equation 4.17, the resulting correction factor is 0.66. This means that the expected settlement will be:

$$S_{field} = \mu \cdot S_{lab} = 0.66 \times 1.30m = .86m$$

At positions other than the centerline, the stress changes and hence, the correction factor, will be slightly different.

<u>Time-Rate of Settlement</u>

When the magnitude of consolidation settlement is large enough to be of concern, it is worthwhile to predict how much of this settlement will occur during the service life

Table 4.4 Calculation of Correction Factor μ - Example 4.2

Depth Interval (meters)	Depth (meters)	$\Delta\sigma_v$ (kPa)	$\Delta\sigma_1$ (kPa)	$\Delta\sigma_2$ (kPa)	$\Delta\sigma_3$ (kPa)	$\Delta\sigma_1+\Delta\sigma_2+\Delta\sigma_3$ (kPa)
3.048 - 6.096	4.572	238.0	282.0	200.7	119.4	602
6.096 - 9.144	7.620	233.8	257.4	151.4	45.5	454.3
9.144 - 12.192	10.668	227.0	245.8	140.7	35.6	422.1
12.192 - 15.240	13.716	218.3	233.2	130.6	28.1	391.9
15.240 - 18.288	16.764	208.6	220.4	121.3	22.2	364.0

$\Sigma\, \Delta\sigma_v = 1125.8$

$\Sigma\, (\Delta\sigma_1+\Delta\sigma_2+\Delta\sigma_3) = 2234.3$

$$\mu = \frac{\frac{1}{3}\,\Sigma[(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3)\Delta z]}{\Sigma(\Delta\sigma_v\, \Delta z)} = \frac{\frac{1}{3}(2234.3)(3.048)}{(1125.8)(3.048)} = 0.66$$

of the embankment. For a homogeneous soil layer beneath a long linear embankment, consolidation occurs due to the dissipation of excess pore pressures in the vertical direction as well as in the horizontal direction parallel to the cross section of the embankment. The governing equation for such a condition is:

$$c_v \frac{\delta^2 u}{\delta z^2} + c_h \frac{\delta^2 u}{\delta x^2} = \frac{\delta u}{\delta t} \tag{4.18}$$

where

c_v = coefficient of consolidation in the vertical direction

c_h = coefficient of consolidation in the horizontal direction

u = excess pore pressure

x, z = the horizontal and vertical coordinate directions

t = time after the excess pore pressures were created

The solution for equation 4.18 depends on the distribution of the excess pore pressure. Therefore, an exact solution is not possible for the general case. An approximate solution is possible, however, by specifying the distribution of pore pressure on a grid of evenly spaced points in the consolidating layer (Figure 4.4). By replacing the partial derivations in equation 4.18 with finite difference approximations on this grid, it is possible to derive the following

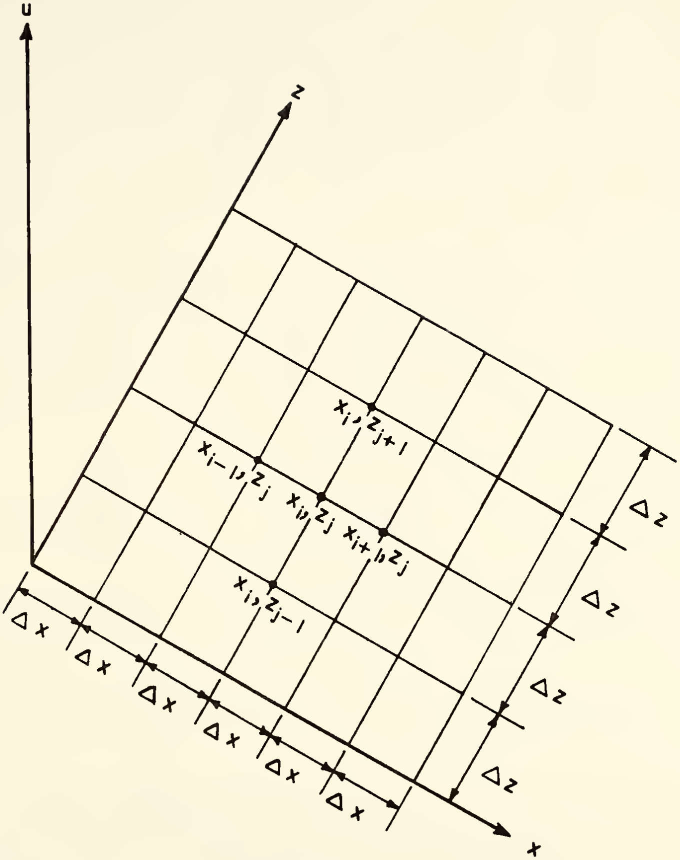


FIGURE 4.4 FINITE DIFFERENCE GRID FOR TWO-DIMENSIONAL CONSOLIDATION EQUATION

finite difference solution for the two-dimensional consolidation equation:

$$u_{i,j,k+1} = \alpha_x [u_{i+1,j,k} + u_{i-1,j,k}] + \alpha_z [u_{i,j+1,k} + u_{i,j-1,k}] + [1 - 2\alpha_x - 2\alpha_z] u_{i,j,k} \tag{4.19}$$

where

i, j are column and row identifiers of the nodal points on the grid illustrated in Figure 4.4

k is the number of the time step

$$\alpha_x = \frac{c_h \Delta t}{(\Delta x)^2} \qquad \alpha_z = \frac{c_v \Delta t}{(\Delta z)^2}$$

$\Delta x, \Delta z$ are the horizontal and vertical spacings of the grid shown in Figure 4.4.

Δt is the time increment used in the analysis.

It is not common practice to evaluate equation 4.19 because c_h is not usually measured. Therefore, it is generally assumed that consolidation occurs solely due to vertical drainage. The governing equation for this condition is the well-known Terzaghi one-dimensional consolidation equation (Terzaghi, 1943)

$$c_v \frac{\delta^2 u}{\delta z^2} = \frac{\delta u}{\delta t} \tag{4.20}$$

The finite difference expression for the solution of this equation is

$$u_{i,k+1} = \alpha_z u_{i+1,k} + (1 - 2\alpha_z) u_{i,k} + \alpha_z u_{i-1,k} \quad (4.21)$$

The accuracy of this expression is maximized when Δz approaches zero and when $\alpha_z = 1/6$ (Perloff and Baron, 1976).

At the boundary of the grid, equation 4.21 must be modified to account for drainage conditions. When a boundary is drained, the excess pore pressure is assumed to have an ambient value at the onset of consolidation equal to half of the initial excess pore pressure. Thereafter, the excess pore pressure at the drained boundary is set equal to zero. To calculate the excess pore pressure at an undrained boundary, it is necessary to assume a "mirror" node just outside the grid with a value of excess pore pressure equal to that of a node just inside the grid. The resulting expressions for nodes at the top and bottom boundary respectively are:

$$u_{i,k+1} = 2\alpha_z \cdot u_{i+1,k} + (1 - 2\alpha_z) \cdot u_{i,k} \quad (4.21a)$$

and

$$u_{i,k+1} = 2\alpha_z \cdot u_{i-1,k} + (1 - 2\alpha_z) \cdot u_{i,k} \quad (4.21b)$$

When the consolidating layer is composed of contiguous soil layers with different values of c_v, continuity of flow

must be satisfied across the layer interfaces. Invoking Darcy's law, the value of the pore pressure at the layer interface may be obtained with the following expression (Harr, 1966):

$$u_{i,k} = u_{i+1,k} - \frac{u_{i+1,k} - u_{i-1,k}}{1 + (k_2/k_1)/(\Delta z_1/\Delta z_2)} \qquad (4.22)$$

where

k_1, k_2 are the permeabilities of the soil above and below the layer interface, respectively

$\Delta z_1, \Delta z_2$ are the grid spacing above and below the layer interface, respectively.

As a first approximation, the ratio of the permeabilities between the upper and lower layers may be taken to be:

$$k_1/k_2 = c_{v1}/c_{v2} \qquad (4.23)$$

where c_{v1} and c_{v2} are the coefficients of consolidation in the upper and lower soil layers, respectively.

Once the excess pore pressure is calculated at all the nodes for a given time after the onset of consolidation, the percent consolidation at that time may be calculated by evaluating the following expression:

$$U\% = \left[1 - \frac{\int_0^H u_i(z) \cdot dz}{\int_0^H u(z) \cdot dz}\right] \times 100 \qquad (4.24)$$

where

$u_i(z)$ = the initial distribution of excess pore pressure with depth

$u(z)$ = the distribution of excess pore pressure with depth at the time in question

H = the thickness of the consolidating layer

z = coordinates in the direction of the depth

Equation 4.24 can be used to calculate the percent consolidation in each layer of a series of continguous soil layers as well as the overall percent consolidation. A program for performing these computations is provided in Appendix H.

Values of the initial excess pore pressure that are to be used in the analysis should be calculated with equation 4.9. This insures that the distribution of excess pore pressures used to calculate the percent consolidation is the same as the distribution used to calculate the magnitude of consolidation settlement.

The coefficient of consolidation is defined by the following relationship:

$$c_v = \frac{k(1 + e)}{a_v \gamma_w} \qquad (4.25)$$

where γ_w is the unit weight of water.

Since the values of the permeability, the void ratio, and the vertical compressibility of the soil change as the consolidation progresses, it is necessary to simplify the characterization of the value of c_v during consolidation. This can be done with the controlled gradient consolidation test which is used in developing the preconsolidation pressure profile. The value of c_v obtained in a controlled gradient test is calculated with the following expression (Lowe, Jonas and Obrician, 1969):

$$c_v = \frac{\delta\sigma}{\delta t}\,\frac{H^2}{2u} \tag{4.26}$$

where

$\frac{\delta\sigma}{\delta t}$ = time rate of change of applied stress

H = the sample thickness

u = excess pore pressure maintained at the undrained end of the sample

This expression allows the calculation of c_v continuously during consolidation without recourse to either the logarithm or square root of time curve fitting methods. Other advantages of this testing procedure include:

1) the test may be run at low strain rates that approach consolidation rates in the field
2) the excess pore pressure is approximately constant across the sample
3) secondary compression does not occur

Once the value of c_v of a soil layer is known along a compression curve like the one shown in Figure 4.1a, it is possible to choose a representative value of c_v corresponding to the average value of vertical effective stress during consolidation. As was the case for calculating the magnitude of settlement, this procedure should be repeated in a number of artificial strata within the soil layer because the soil parameters and the stress changes will vary with depth.

Budget and time constraints may prohibit the type and amount of testing necessary to perform the analysis just described. When this is the case, an estimate of the variation of c_v with depth needs to be made. In general, the value of c_v in a soil that is overconsolidated will be substantially higher than c_v of a normally consolidated sample of the same soil. If it is possible to estimate which portions of the soil layer will be normally consolidated and overconsolidated during the consolidation process, the entire layer can be reduced to a two-strata system. Normally, c_v of the upper portion will correspond to an overconsolidated state and c_v of the lower portion will correspond to a normally consolidated state.

Example 4.3

It is desired to estimate the time rate of settlement of the consolidation settlement at the centerline of the

embankment in Example 4.1. The liquid limit of the soil in the consolidating layer is 45. Typical values of c_v are 8.61 m^2/day (0.8 ft^2/day) and 2.15 m^2/day (0.2 ft^2/day) for the overconsolidated and normally consolidated portions of the layer, respectively (Holtz and Kovacs, 1981). The initial excess pore pressures were estimated using equation 4.10. The values of the stress changes were taken from Table 4.1. It was assumed that the overconsolidated portion occupies the upper 6.1m (20 ft) of the soil layer. In this example the bottom of the consolidating layer is undrained. The results, obtained with the computer program in Appendix H, are shown in Table 4.5. The time rate of settlement in the normally consolidated stratum proceeds at a much slower rate than the rate of the entire layer. Since most of the settlement occurs in the normally consolidated stratum, it would be unconservative to estimate the time rate of settlement on the basis of the overall rate of consolidation for the soil layer. A better estimate of the time rate in this case can be made by assuming that all of the settlement occurs in the normally consolidated stratum and proceeds at the rate computed for this stratum. This calculation is included in Table 4.5.

Table 4.5 Percent Consolidation of Clay Layer - Example 4.3

	Percent Consolidation (%)			
Time (days)	Over Consolidated Strata	Normally Consolidated Strata	Entire Layer	Centerline Settlement (meters)
20	25.3	0.0	11.4	0.0000
60	43.8	0.0	20.6	0.0000
120	60.6	2.1	30.2	.0183
230	75.8	8.2	40.6	.0704
420	85.5	17.3	50.0	.1484
820	91.2	31.5	60.1	.2694
1480	94.0	48.0	70.0	.4109
2480	96.1	65.3	80.0	.5590
4200	98.0	82.6	90.0	.7074
5920	99.0	91.3	95.0	.7818

V - CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

Conclusions

1. A hybrid method of compaction specification which makes the insitu water content of the embankment soil equal to the OMC is introduced.
2. The Simplified Bishop factor of safety option in the STABL program has been recoded to correct various difficulties.
3. A methodology for adjusting the Simplified Janbu factor of safety that is used by several STABL options to definitions of the factor of safety that are more familiar is presented.
4. Embankment side slope design has been illustrated for short and long term situations using laboratory compacted shear strength data.
5. Geometric and probabilistic interpretations of the factor of safety are introduced to illustrate their usefulness in the selection of an appropriate factor of safety for design.
6. A methodology of predicting settlement of embankment foundations has been illustrated. Computer programs

to compute the magnitude and time-rate of settlement are included. These programs, in conjunction with STABL, form an analysis package for the design of embankments.

Recommendations

1. To use the hybrid approach for specifying compaction, values of the coefficient of compaction that reflect the influence of soil type, compactor type, the operating procedure, and the number of passes need to be developed.
2. The current version of STABL should be augmented by adding a complete equilibrium method of calculating the factor of safety such as the Spencer method (Spencer, 1973).
3. The LEMIX program, which calculates the factor of safety on a three-dimensional surface, demonstrates the importance of three-dimensional effects. The next step in this field should be the development of a program that allows the random generation of general shaped three-dimensional surfaces as well as the subsequent calculation of the factor of safety on these surfaces.
4. The geometric and probabilistic approaches which were introduced should be further developed to determine acceptable design values of the geometric factor of

safety and the probability of failure for use in slope design.

5. Procedures should be developed to apply the statistical relationships between the material parameters of lab and field compacted clays in settlement predictions within an embankment and in slope stability analysis.

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REFERENCES

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APPENDICES

APPENDIX A

DERIVATION OF THE SIMPLIFIED BISHOP FACTOR OF SAFETY

Notation after Siegel (1975).

STEP 1 - Enforce moment equilibrium of sliding circular mass divided into n slices:

$$\Sigma M_o = 0$$

$$= \sum_1^n [(\Delta W \cdot (1-k_v) + \Delta U_\beta \cdot \cos\beta + \Delta Q \cdot \cos\delta)(R \cdot \sin\alpha)]$$

$$- \sum_1^n [\Delta S_r] \cdot R - \sum_1^n [(\Delta U_\beta \cdot \sin\beta + \Delta Q \cdot \sin\delta)(R \cdot \cos\alpha - h)]$$

$$+ \sum_1^n [K_h \cdot \Delta W (R \cdot \cos\alpha - h_{eq})] \qquad (A.1)$$

where

R = radius of the circle

$$\Delta S_r = \frac{1}{FS} [C'_a + \Delta N' \cdot \tan\phi'_a] \qquad (A.2)$$

$$C'_a = c \cdot dx/\cos\alpha$$

Dividing equation A.1 by R yields:

$$\Sigma M_o = \sum_1^n [(\Delta W'(1-k_v) + \Delta U_\beta' \cos\beta + \Delta Q' \cos\delta)' \sin\alpha]$$
$$- \sum_1^n [\Delta S_r] - \sum_1^n [(\Delta U_\beta' \sin\beta + \Delta Q \sin\delta)(\cos\alpha - h/R)$$
$$+ \sum_1^n [k_h' \Delta W' (\cos\alpha - \frac{h_{eq}}{R})] \qquad (A.3)$$

STEP II - Substitute ΔS_r into equation A.3 and assume the factor of safety is equivalent in each slice:

$$FS = \frac{\sum_1^n (C'_a + \Delta N' \tan\phi'_a)}{\sum_1^n A_3 - \sum_1^n A_4 + \sum_1^n A_5} \qquad (A.4)$$

where

$$A_3 = (\Delta W'(1-k_v) + \Delta U_\beta' \cos\beta + \Delta Q' \cos\delta)' \sin\alpha \qquad (A.4a)$$
$$A_4 = (\Delta U_\beta' \sin\beta + \Delta Q' \sin\delta)(\cos\alpha - h/R) \qquad (A.4b)$$
$$A_5 = k_h' \Delta W (\cos\alpha - \frac{h_{eq}}{r}) \qquad (A.4c)$$

STEP III - Sum forces in vertical direction for each slice:

$$\Sigma F_v = \Delta W'(1-k_v) - (C'_a + \Delta N'' \tan\phi'_a)' \sin\alpha/FS - \Delta N'' \cos\alpha$$
$$+ \Delta Q' \cos\delta + \Delta U_\beta' \cos\beta - \Delta U_\alpha' \cos\alpha \qquad (A.5)$$

Rearranging equation A.5 yields:

$$\Delta N''(\tan\phi'_a' \sin\alpha/FS + \cos\alpha) =$$
$$\Delta W(1-k_v) - C'_a' \sin\alpha/FS + \Delta Q' \cos\delta + \Delta U_\beta' \cos\beta - \Delta U_\alpha' \cos\alpha \qquad (A.6)$$

Finally,

$$\Delta N' = \frac{\Delta W(1-k_v) - C'_a \cdot \sin\alpha/FS + \Delta Q \cdot \cos\delta + \Delta U_\beta \cdot \cos\beta - \Delta U_\alpha \cdot \cos\alpha}{\cos\alpha + \tan\phi'_a \cdot \sin\alpha/FS} \qquad (A.7)$$

Substituting equation A.7 into equation A.4 yields:

$$FS = [C'_a + \tan\phi'_a \cdot (\Delta W(1-k_v) - C'_a \cdot \sin\alpha/FS$$
$$+ \Delta Q \cdot \cos\delta + \Delta U_\beta \cdot \cos\beta - \Delta U_\alpha \cdot \cos\alpha)/$$
$$(\cos\alpha + \tan\phi'_a \cdot \sin\alpha/FS]/(\Sigma A_3 - \Sigma A_4 + \Sigma A_5) \qquad (A.8)$$

or rearranging

$$FS = \frac{\sum_1^n \left[\frac{C'_a + \tan\phi'_a \sec\alpha(\Delta W(1-k_v) + \Delta Q\cos\delta + \Delta U_\beta\cos\beta - \Delta U_\alpha\cos\alpha)}{1 + \tan\phi'_a \cdot \tan\alpha/FS}\right]}{\sum_1^n A_3 - \sum_1^n A_4 + \sum_1^n A_5} \qquad (A.9)$$

For simplicity of coding equation A.9 may be written as follows:

$$FS = \frac{\sum_1^n \left[\frac{A_1}{1 + A_2/FS}\right]}{\sum_1^n A_3 - \sum_1^n A_4 + \sum_1^n A_5} \qquad (A.10)$$

where

$$A_1 = C'_a + \tan\phi'_a \cdot \sec\alpha(\Delta W(1-k_v) + \Delta Q \cdot \cos\delta \qquad (A.10a)$$
$$+ \Delta U_\beta \cos\beta - \Delta U_\alpha \cdot \cos\alpha)$$

$$A_2 = \tan \phi'_a \cdot \tan\alpha \qquad \text{(A.10b)}$$

This expression is programmed in STABL3.

The changes in the STABL code necessary to program equation A.10 are included in the following pages.

Changes to the STABL Program

Changes to the Main Program:

```
c                                                      stbl [illegible]
   11 ireac=0                                          stbl 308
      read(5,100,end=19) mkeyw                         mgoodman
  100 format(a6)                                       stbl 312
c                                                      stbl 326
c     ---------------------------------                stbl 328
c     check if profil is initial command               stbl 330
c     ---------------------------------                stbl 332
c                                                      stbl 334
      if(iprof.eq.1)go to 23                           mgoodman
      if(mkeyw.eq.keyw(1))go to 24                     stbl 338
```

Changes to Subroutine SURFAC:

```
c                                                                        surf 120
c        radius   distance between the coordinate points and the         mgoodman
c                 center of the limit equilibrium surface (mb=1)         mgoodman
c                                                                        mgoodman
c        reacer   subroutine that reads integer or real data in free     surf 122

c                                                                        surf 136
c        xcntr    x coordinate of the center of a circular limit         mgoodman
c                 equilibrium surface                                    mgoodman
c        xhalf2   x coordinate of the midpoint of the second segment     mgoodman
c                 on the limit equilibrium surface                       mgoodman
c                                                                        mgoodman
c        x1       x coordinate of the first point on the limit           mgoodman
```

```
c                   equilibrium surface                              mgoodman
c                                                                    mgoodman
c        x2         x coordinate of the second point on the limit    mgoodman
c                   equilibrium surface                              mgoodman
c                                                                    mgoodman
c        x3         x coordinate of the third point on the limit     mgoodman
c                   equilibrium surface                              mgoodman
c                                                                    mgoodman
c        ycntr      y coordinate of the center of the limit equilibrium mgoodman
c                   surface                                          mgoodman
c                                                                    mgoodman
c        yhalf2     y coordinate of the midpoint of the second segment mgoodman
c                   on the limit equilibrium surface                 mgoodman
c                                                                    mgoodman
c        y1         y coordinate of the first point on the limit     mgoodman
c                   equilibrium surface                              mgoodman
c                                                                    mgoodman
c        y2         y coordinate of the second point on the limit    mgoodman
c                   equilibrium surface                              mgoodman
c                                                                    mgoodman
c        y3         y coordinate of the third point on the limit     mgoodman
c                   surface
c  ------------------------------------------------------------------surf 138


      common /blk03/nsurf,surf(100,2)                                 surf 152
      common /blk15/ m,mh                                             mgoodman
      common /blk20/ radius                                           mgoodman
      dimension error(6)                                              surf 154
```

```
  103 format(12x,i3,2x,2f12.2)                                            surf 412
c                                                                         mgoodman
c      calculate the radius of the limit equilibrium surface if the       mgoodman
c      the simplified bishop method is being used                         mgoodman
c                                                                         mgoodman
      if (mb.ne.1) go to 104                                              mgoodman
      x1 = surf(1,1)                                                      mgoodman
      y1 = surf(1,2)                                                      mgoodman
      x2 = surf(2,1)                                                      mgoodman
      y2 = surf(2,2)                                                      mgoodman
      x3 = surf(3,1)                                                      mgoodman
      y3 = surf(3,2)                                                      mgoodman
      xhalf2 = (x2 + x3)/2.0                                              mgoodman
      yhalf2 = (y2 + y3)/2.0                                              mgoodman
      xcntr = ((x1**2-x2**2)*(y3-y2) - (x2**2-x3**2)*(y2-y1) + (y3-y1)   mgoodman
     1          *(y2-y1)*(y3-y2))/(2.0*((x1-x2)*(y3-y2) - (x2-x3)*       mgoodman
     2          (y2-y1)))                                                 mgoodman
      ycntr = (x2-x3)/(y3-y2)*(xcntr - xhalf2) + yhalf2                   mgoodman
      radius = sqrt((xcntr - x2)**2 + (ycntr - y2)**2)                    mgoodman

      print,'            radius = ',radius,' feet'                        mgoodman
  104 return                                                              mgoodman
      end                                                                 surf 416
```

Changes to Subroutine RANSUF:

```
                                                                     rans 253
c                                                                    rans 254
c       mb       control variable that indicates whether the         mgoodman
c                simplified bishop or the simplified janbu is to be  mgoodman
c                mb = 1     if the simplified bishop method is used  mgoodman
c                                                                    mgoodman
c       n        index variable for array subscripting.              jul76ebo
```

```
c                                                                    rans 314
c             radius   the radius of the points on the simplified mgoodman
c                      bishop limit equilibrium surface              mgoodman
c                                                                    mgoodman
c       ranf     function subprogram that generates a pseudo-random  rans 320
```

```
   22 ctheta=thetae+(thetab-thetae)*ranf(0.)**2                      rans 958
c                                                                    mgoodman
c     calculate the radius of the limit equiblibrium surface if the  mgoodman
c     simplified bishop factor of safety is being employed           mgoodman
c                                                                    mgoodman
      if (mb.eq.1) radius = tsurf/2.0/sin(dtheta/2.0)                mgoodman
      theta=thetas                                                   rans 959
```

Changes to Subroutine WEIGHT:

```
c                                                                        wght 112
c       hight     array containing the hights of the slices              mgoodman
c                                                                        mgoodman
c       hghteq    array of the values of the hight of the centroid of    mgoodman
c                 the horizontal earthquake forces above the base of     mgoodman
c                 each slice                                             mgoodman
c                                                                        mgoodman
c       i         incex variable for array subscripting.                 wght 114

c                                                                        wght 216
c       mb        variable use to descrimitnate between the simplified   mgoodman
c                 bishop arc the simplified janbu methods                mgoodman
c                 mb = 1     if the simplifiec bishop method is used     mgoodman
c                                                                        mgoodman
c       n         incex variable for array subscripting.                 wght 218

c                                                                        wght 340
c       wtheq     the procuct of the weight of a slice and the distancemgoodman
c                 between its base and the centroid of its horizontal    mgoodman
c                 earthcuake force                                       mgoodman
c                                                                        mgoodman
c       wt        weight cf a slice subsection.                          wght 342

      common /blk11/cavt,kccef,vkcoef
      common /blk15/m,mb                                                 wght 422
      common /blk21/hight(200),hghteq(200)                               mgoodman
      dimension yi(20),yw(20),jw(10),y(10)                               mgoodman
                                                                         wght 424
```

```
      js=1                                                                wght 434
      if (rpz.eq.0) go to 110                                             mgoodman
      do 13 j=1,npz                                                       wght 436
      jw(j)=2                                                             wght 438
   13 continue                                                            wght 440
  110 continue                                                            mgoodman
      rtop1=rtop+1                                                        wght 442

      do 2 i=1,nslice                                                     wght 454
c                                                                         mgoodman
c     initialize the slice hight and the hight of the centroid of the    mgoodman
c     earthquake force above the base of the slice                        mgoodman
c                                                                         mgoodman

      hight(i) = 0.0                                                      mgoodman
      hghteq(i) = 0.0                                                     mgoodman
      dx(i)=x(i+1)-x(i)                                                   wght 456

c                                                                         wght 676
   10  if (mb.eq.1) wtheq = 0.0                                           mgoodman
       if(k.eq.1)go to 14                                                 mgoodman
```

```
      call scilwt(yt,yi(1),itp(jt),yw(1),wtt(i),i)                       wght 680
      if (mb.ec.1) wtheq = wtheq + ((yt + yi(1))/2.0 - yb)*wtt(i)         mgoodman
      cc tc 15                                                           wght 682
   14 call scilwt(yt,yb,itp(jt),yw(1),wtt(i),i)                          wght 684
      if (mb.ec.1) wtheq = wtheq + (yt-yb)/2.0*wtt(i)                    mgoodman
      gc tc 21                                                           wght 686
   15 if(k.ec.2)go to 17                                                 wght 688
      k1=k-1                                                             wght 689
      cc 11 j=2,k1                                                       wght 690
      call scilwt(yi(j-1),yi(j),soil(j),yw(j),wt,i)                      wght 692
      if (mb.eq.1) wtheq = wtheq + ((yi(j-1) + yi(j))/2.0 - yb)*wt       mgoodman
      wtt(i)=wtt(i)+wt                                                   wght 694
   11 ccrtinue                                                           wght 696
   17 call soilwt(yi(k-1),yb,soil(k),yw(k),wt,i)                         wght 698
      if (mb.ec.1) wtheq = wtheq + (yi(k-1) - yb)/2.0*wt                 mgoodman
      wtt(i)=wtt(i)+wt                                                   wght 700


      celt(i)=0.                                                         wght 716
      if(isurc.ec.0)gc to 120                                            mgoodman
c                                                                        wght 720


   20 js=jtn                                                             wght 746
  120 if (mb.re.1) go to 2                                               mgoodman
c                                                                        mgoodman
c     calculate the (1) the slice hight and (2) the hight of the         mgoodman
c     certroic of the horizontal earthquake force component above the    mgoodman
c     base of the slice                                                  mgoodman
c                                                                        mgoodman
      hight(i) = yt - yb                                                 mgoodman
      hghtec(i) = wthec/wtt(i)                                           mgoodman
    2 ccrtinue                                                           wght 748
```

Changes to subroutine FACTR:

```
c                                                                 fctr  64
c        a1        array usec in factor of safety calculation.    mgoodman
c                                                                 fctr  68
c        a2        array usec in factor of safety calculation.    mgoodman
c                                                                 fctr  72
c        a3        array used in factor of safety calculation.    mgoodman
c                                                                 fctr  76
c        a4        term usec in factor of safety calculation      mgoodman


c                                                                 mgoooman
c        a5        term usec in factor of safety calculation      mgoodman
c                                                                 mgoodman
c        beta      array ccntaining values of the angle of the top of   fctr  78


c                                                                 fctr 242
c        radius    the length of the radius of the points on the  mgoodman
c                  simplifiec bishop limit equilibrium surface    mgoodman
c                                                                 mgoodman
c        rd        factor fcr conversion of degrees to radians.   fctr 244

      ccmmon /blk15/ m,mb                                         fctr 363
      ccmmon /blk20/radius                                        mgoodman
      ccmmcn /blk21/hicht(200),hghteq(200)                        mgoodman
      cimension a1(20C),a2(200),a3(200)                           fctr 364
```

```
c                                                                        fctr 458
      if (mb.eq.1) go to 40                                              mgoodman
c                                                                        mgoodman
c     simplified janbu a-terms                                           mgoodman
c                                                                        mgoodman
      a0 = cslice*cx(i) + tp*(wtt(i)*(1.0-vkcoef) - ualpha(i)*ca         fctr 459
     1      + ubeta(i)*cb + p(i)*cd)                                     fctr 460
      a1(i) = a0/ca**2                                                   fctr 462
      a2(i)=wtt(i)*(ta+kcoef-vkcoef*ta)+ubeta(i)*(cb*ta-sb)+p(i)*(cd*ta-fctr 464
     1   sd)                                                             fctr 466
      a3(i)=ta*tp                                                        fctr 468
      sumb = sumb + a2(i)                                                fctr 469
      go to 2                                                            mgoodman
c                                                                        mgoodman
c     simplified bishop a-terms                                          mgoodman
c                                                                        mgoodman
40    a1(i) = cslice*cx(i)/ca + tp/ca*(wtt(i)*(1.0-vkcoef) + p(i)*cd +   mgoodman
     1         ubeta(i)*cb - ualpha(i)*ca)                               mgoodman
      a2(i) = tp*ta                                                      mgoodman
      a3(i) = (wtt(i)*(1.0-vkcoef) + ubeta(i)*cb + p(i)*cd)*sa           mgoodman
      a4 = (ubeta(i)*sb + p(i)*sd)*(ca-hight(i)/radius)                  mgoodman
      a5 = kcoef*wtt(i)*(ca-hghteq(i)/radius)                            mgoodman
      sumb = sumb + a3(i) - a4 + a5                                      mgoodman
    2 continue                                                           fctr 470


     if (mb.eq.1) go to 50                                             mgoodman
     sumt = a1(i)/(1.0 + a3(i)/fold) + sumt                            fctr 496


      go to 6                                                            mgoodman
50    sumt = sumt + a1(i)/(1.0 + a2(i)/fold)                             mgoodman
    6 continue                                                           fctr 500
```

APPENDIX B

CALCULATION OF THE RADIUS OF A SPECIFIED CIRCULAR SURFACE

STABL randomly generates circular surfaces by generating successive chords of a circle which are inclined at a deflection angle with one another. Since all of the chords are of equal length, the radius may be determined with the following equation:

$$R = \frac{T}{2} \sin \left(\frac{\Delta\theta}{2}\right) \qquad (B.1)$$

where

R = radius of the circle

T = chord length of the segments circumscribing the circle

$\Delta\theta$ = deflection angle of the segments circumscribing the circle

However, when a specified surface is input, the chord lengths and deflection angles will not have constant values. In this case a more general expression for calculating the radius is required. To do this any two adjacent segments on a known circular surface may be taken (Figure B.1).

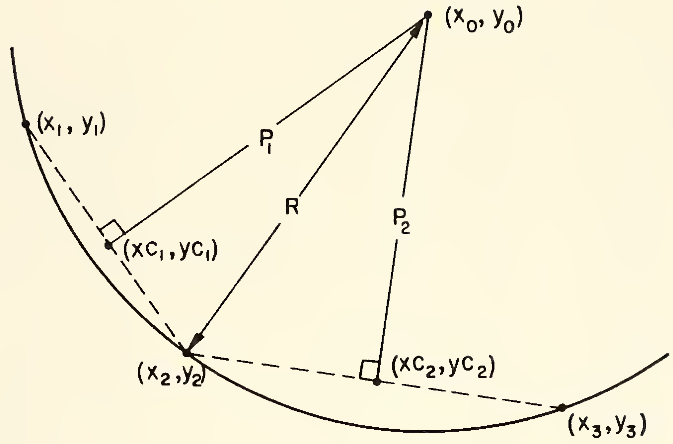

FIGURE B.1 GEOMETRY OF FIRST 2 CHORDS OF A SPECIFIED CIRCULAR SURFACE

The equation of line P1 which is the perpendicular bisector of the chord defined by (x_1,y_1) and (x_2,y_2) is

$$y = -\frac{(x_2-x_1)}{(y_2-y_1)}(x-x_{c1}) + y_{c1} \tag{B.2}$$

where

(x_1,y_1) are the coordinates of the first point on the first chord

(x_2,y_2) are the coordinates of the second point on the first chord and the first point on the second chord

(x_{c1},y_{c1}) are the coordinates of the intersection of line P1 and the first chord.

Similarly, the equation of line P2, which is the perpendicular bisector of the chord defined by (x_2,y_2) and (x_3,y_3), is

$$y = -\frac{x_3-x_2}{y_3-y_2}(x-x_{c2}) + y_{c2} \tag{B.3}$$

where

(x_3,y_3) are the coordinates of the second point on the second chord

(x_{c2},y_{c2}) are the coordinates of the intersection of line P2 and the second chord.

Lines P1 and P2 intersect at the center of the circle. The coordinates of the center of the circle may be found by

setting equations B.2 and B.3 equal to each other. The resulting expression is:

$$\frac{x_1-x_2}{y_2-y_1}(x_o-\frac{x_1+x_2}{2}) + \frac{y_1+y_2}{2} = \frac{x_2-x_3}{y_3-y_2}(x_o - \frac{x_2+x_3}{2}) + \frac{y_2+y_3}{2} \quad (B.4)$$

Rearranging,

$$x_o = [(x_1^2-x_2^2) - (x_2^2-x_3^2)(y_2-y_1) + (y_3-y_1)(y_2-y_1)(y_3-y_2)]/2/ \\ [(x_1-x_2)(y_3-y_2) - (x_2-x_3)(y_2-y_1)] \quad (B.5)$$

The value of y_o may be obtained by substituting the value of x_o in equation B.3. The result is

$$y_o = \frac{x_2-x_3}{y_3-y_2}(x_o-x_{c2}) + y_{c2} \quad (B.6)$$

where

$$x_{c2} = (x_2+x_3)/2$$
$$y_{c2} = (y_2+y_3)/2$$

Finally, the radius may be found with the following expression:

$$R = \sqrt{(x_o-x_2)^2 + (y_o-y_2)^2} \quad (B.7)$$

These equations are coded in the current version of the STABL program.

APPENDIX C

THE FRICTION CIRCLE FACTOR OF SAFETY

The stability number, $c/(FS \cdot \gamma \cdot H)$, of a trial circular surface through a homogeneous slope such as the one shown in Figure C.1 may be determined by the following sequence of calculations (Taylor, 1940):

$$n = \frac{1}{2}\,(\cot x - \cot y - \cot \beta + \sin\phi \cdot \csc x \cdot \csc y) \qquad (C.1)$$

where

β = sideslope

ϕ = friction angle of the slope

x, y = angles shown in Figure C.1

If $n > 0$, the trial circle passes under the toe of the slope as shown by surface III in Figure C.2. If $n \leq 0$, the trial surface starts at the toe as shown by surface I or II in Figure C.2.

When $n \leq 0$, equations C.2 through C.5 are evaluated.

$$\frac{H}{2d} = \frac{\frac{1}{2}\csc^2 x(y \cdot \csc^2 y - \cot y) + \cot x - \cot\beta}{\frac{1}{3}(1-2\cot^2\beta) + \cot\beta(\cot x - \cot y) + \cot x \cdot \cot y} \qquad (C.2)$$

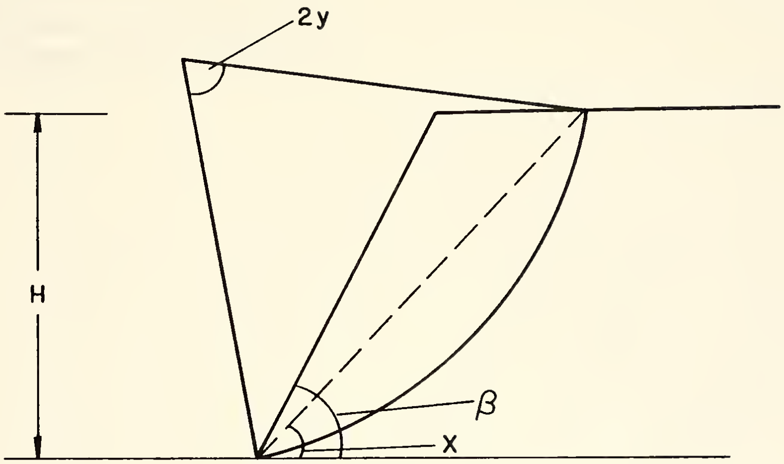

FIGURE C.1 GEOMETRY REQUIRED TO SPECIFY A CIRCULAR SLIP SURFACE FOR A SIMPLE SLOPE

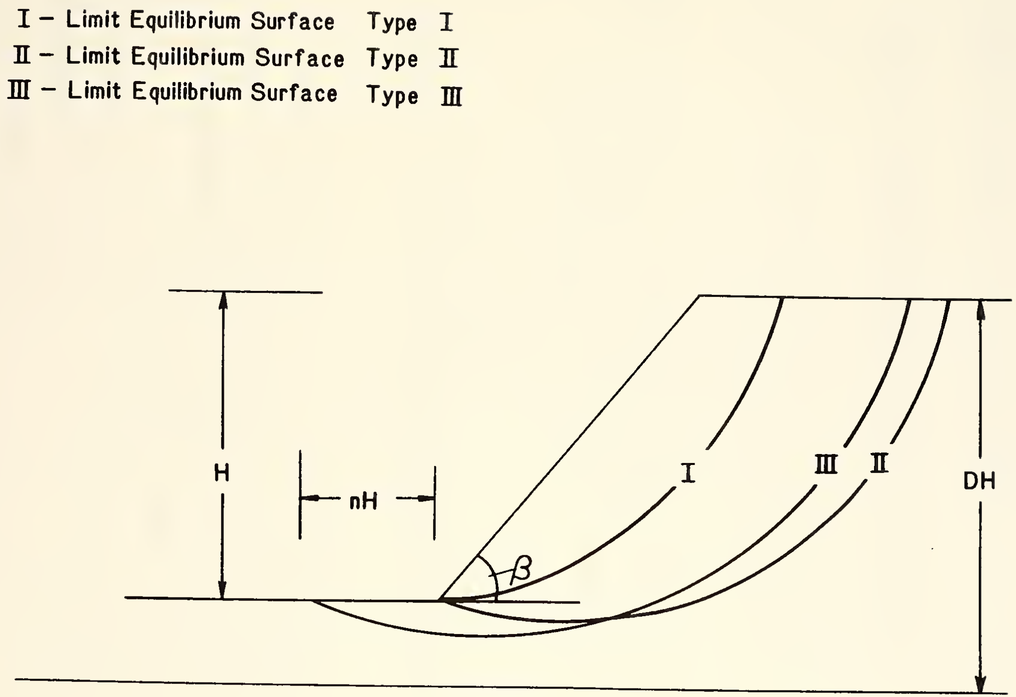

FIGURE C.2

ILLUSTRATION OF TOE FACTOR AND DEPTH FACTOR OF A SIMPLE SLOPE

$$\cot u = \frac{H}{2d} \cdot y \cdot \sec x \cdot \csc x \cdot \csc^2 y - \tan x \tag{C.3}$$

$$\sin(u-v) = \frac{H}{2d} \cdot \sin u \cdot \csc x \cdot \csc y \cdot \sin\phi \tag{C.4}$$

$$\frac{c}{FS \cdot \gamma \cdot H} = \frac{\frac{1}{2}\csc^2 x(y\csc^2 y - \cot y) + \cot x - \cot\beta}{2\cot x \cdot \cot v + 2} \tag{C.5}$$

When $n > 0$, equations C.6 and C.7 replace equations C.2 and C.5.

$$\begin{aligned}\frac{H}{2d} = &[\tfrac{1}{2}\csc^2 x \cdot (y \cdot \csc^2 y - \cot y) + \cot x - \cot\beta - 2n]/\\ &[\tfrac{1}{3}(1-2\cot^2\beta) + \cot\beta \cdot (\cot x - \cot y) + \cot x \cdot \cot y\\ &+ 2n^2 - 2n \cdot \sin\phi \cdot \csc x \cdot \csc y]\end{aligned} \tag{C.6}$$

$$\frac{c}{FS \cdot \gamma \cdot H} = \frac{\frac{1}{2}\csc^2 x(y \cdot \csc^2 y - \cot y) + \cot x - \cot\beta - 2n}{2\cot x \cdot \cot v + 2} \tag{C.7}$$

Given β and ϕ, it is possible to search for the surface on which the stability number has a maximum value by checking all combinations of the angle x from 0 up to β and all the values of the angle y from 0 to 90°. The search may be limited to $y \leq x$ if the limit equilibrium surface does not go beneath the elevation of the toe. The value of the stability number so obtained is almost identical to that which is presented in the stability chart that Taylor developed for simple slopes in homogeneous material except that the

equations presented here do not reflect the correction for the error associated with the assumption that the resultant of the normal and frictional forces acts tangent to the friction circle. The maximum value of this correction is approximately 7% (Taylor, 1937).

To calculate the factor of safety of a slope, the factor of safety must be the same on the cohesion intercept and the friction angle, i.e.,

$$FS_c = FS_\phi \tag{C.8}$$

This condition may be satisfied with the following iterative procedure which is used by the computer program included hereafter:

1. Assume a value of FS_ϕ
2. Calculate the required value of ϕ as follows:

$$\phi_{req} = \tan^{-1} \left(\frac{\tan\phi}{FS_\phi}\right) \tag{C.9}$$

3. Calculate the maximum stability number, N_s, for ϕ_{req} with equation C.7.
4. Calculate FS_c with the expression:

$$FS_c = (c/\gamma H)/N_s \tag{C.10}$$

5. Compare FS_c and FS_ϕ
   a) If $\phi = 0$, then $FS = FS_c$.
   b) If $\phi \neq 0$ and $FS_\phi = FS_c$, then convergence has been obtained.
   c) If $\phi \neq 0$ and $FS_\phi \neq FS_c$, repeat steps 2 through 5.

This procedure works except when the stability numbers close to zero cause numerical instability.

User Manual - Friction Circle Factor of Safety Program

The following program was developed in 77 Fortran on a CDC 6000 series computer. All input is in English units. However, any dimensionally homogeneous set of units will work. All input is unformatted.

1. Input on one record:

   a) the friction angle of the slope (degrees)

   b) the slope angle (degrees)

   c) the cohesion intercept (psf)

   d) the slope height (ft)

   e) the density of the soil (pcf)

2. Input on one record:

   a) an integer variable controlling the limits of the search:

   if the slip circle is completely above the elevation of the toe input '1'

   if the slip surface intersects the toe of the slope and may descend down to a specified depth limit, input '2'

   if the slip surface may pass under the toe of the slope, input '3'

3 If the value of the integer variable on record #2 is '2', input on one record:

   a) the depth factor of the search.

   The depth limit is the depth beneath the top of the slope to the bottom of the deepest circle that is geometrically possible.

The depth factor is obtained by dividing the depth limit by the slope height. The depth limit, DH, is illustrated in Figure D.2.

4. If the value of the integer on record #2 is '3', input on one record:

   a) the depth factor of the search.

   b) the toe factor of the search.

   The toe factor is the multiple of the slopes height that the slip circle can extend beyond the slopes toe (see Figure D.2).

The program listing is included in the following pages.

```
      common/hate/ numx,numy,toe,slope,dangle,
     1               depthf,nlimit,nsmax
      common/lost/ coti,sinphi
      real nsmax,nlimit
      integer toe,cycles
      data tol1,tol2,cycles,fsphi/1.0e-8,0.01,100,1.0/
c
c
c
      pi = acos(-1.0)
c
c
      read (5,*) phi,slope,cohes,hight,gamma
      write (6,10) phi,cohes,gamma,hight,slope
10    format (' friction angle=',f6.1,'degrees',/,
     1        ' cohesion=',f8.1,/,
     2        ' density=',f6.1,/,
     3        ' hight=',f6.1,/,
     4        ' side-slope=',f6.1,' degrees',/)
      read (5,*) toe
      if (toe.eq.1) write(6,12)
12    format (' circles start at the toe with depth factor=1 ',//)
      if (toe.eq.2) read(5,*) depthf
      if (toe.eq.3)  read(5,*) depthf,nlimit
      if (toe.eq.2) write(6,14) depthf
14    format (' D =',f5.1,//)
      if (toe.eq.3) write(6,16) depthf,nlimit
16    format (' D =',depthf,/,' n =',nlimit,//)
c
c
      numx = int(slope) - 1
      numy = 89
      if (toe.eq.1) numy = numx
      slope = slope*pi/180.0
      phi = phi*pi/180.0
      if (abs(slope-pi/2.0).lt.tol1) then
          coti = 0.0
      else
          coti = 1.0/tan(slope)
      endif
      dangle = pi/180.0
      sinphi = sin(phi)
      cgamh = cohes/(gamma*hight)
c
c
      do 100 i = 1,cycles
      phireq = atan(tan(phi)/fsphi)
      sinphi = sin(phireq)
      call phicir
      fscohs = cgamh/nsmax
      if (phi.lt.tol2) then
          print*,'fs=',fscohs
          print*,'stability number=',nsmax
```

```
          stop
      endif
      fsdiff = abs(fscohs-fsphi)
      print*, fsphi, fscohs, nsmax
      if (fsdiff.lt.tol2) then
          print*, ' fs= ', fscohs
          stop
      elseif (fsdiff.ge.tol2) then
          fsphi = (fsphi + fscohs)/2.0
      endif
      if (i.eq.cycles) then
          print*, ' convergence not obtained'
          stop
      endif
100   continue
c
c
      end
c
c
c
      subroutine phicir
      common/hate/ numx,numy,toe,slope,dangle,
     1             depthf,nlimit,nsmax
      common/lost/ coti,sinphi
      common/love/ en,x,y,cscx,cscy,cotx,coty,stbnum
      real nsmax,nlimit
      integer toe
      y = 0.0
      nsmax = 0.0
c
c
      do 25 itery = 1,numy
      y = y + dangle
      coty = 1.0/tan(y)
      cscy = 1.0/sin(y)
      x = 0.0
      do 25 iterx = 1,numx
      x = x + dangle
      if (toe.eq.1.and.x.lt.y) go to 25
      cotx = 1.0/tan(x)
      cscx = 1.0/sin(x)
      en = 0.5*(cotx - coty - coti + sinphi*cscx*cscy)
      if ((toe.eq.1.or.toe.eq.2).and.en.gt.0.0) go to 25
      if (toe.eq.3.and.en.gt.nlimit) go to 25
      d = 0.5*(cscx*cscy - cotx*coty + 1.0)
      if (toe.ne.1.and.d.gt.depthf.and.y.gt.x) go to 25
      call stabnm
      nsmax = amax1(nsmax,stbnum)
25    continue
c
c
      return
```

```
      end
c
c
c
      subroutine stabnm
      common/love/ en,x,y,cscx,cscy,cotx,coty,stbnum
      common/lost/ coti,sinphi
c
c
c     determine the stability number of a specified slip
c     surface on a  specified slope.
c
c
c
      secx = 1.0/cos(x)
      if (en.le.0.0) then
c     toe circle
      param1 = (0.5*cscx**2*(y*cscy**2 - coty) + cotx
     1          - coti)/(1.0/3.0*(1.0 -2.0*coti**2) + coti
     2          *(cotx - coty) + cotx*coty)
      param2 = param1*y*secx*cscx*cscy**2 - tan(x)
      u =  atan(1.0/param2)
      param3 = param1*sin(u)*cscx*cscy*sinphi
      uv = asin(param3)
      v = u - uv
      stbnum = (0.5*cscx**2*(y*cscy**2 - coty) + cotx -
     1          coti)/(2.0*cotx/tan(v) + 2.0)
      elseif (en.gt.0.0) then
c     slip surface beneath the toe of the slope
      param1 = (0.5*cscx**2*(y*cscy**2 - coty) + cotx  -
     1          coti - 2.0*en)/(1.0/3.0*(1.0 - 2.0*coti
     2          **2) + coti*(cotx - coty) + cotx*coty +
     3          2.0*en**2 - 2.0*en*sinphi*cscx*cscy)
      param2 = param1*y*secx*cscx*cscy**2 - tan(x)
      u = atan(1.0/param2)
      param3 = param1*sin(u)*cscx*cscy*sinphi
      uv = asin(param3)
      v = u - uv
      stbnum = (0.5*cscx**2*(y*cscy**2 - coty) + cotx  -
     1          coti - 2.0*en)/(2.0*cotx/tan(v) + 2.0)
      endif
      return
      end
```

APPENDIX D

THE BETA DISTRIBUTION

The beta distribution for the variable x is defined by the following density function (Harr, 1977):

$$f(x) = \frac{1}{b-a} \frac{\Gamma\ (\alpha+\beta+2)}{\Gamma\ (\alpha+1)\ \Gamma\ (\beta+1)} \left(\frac{x-a}{b-a}\right)^{\alpha} \left(\frac{b-x}{b-a}\right)^{\beta} \qquad \text{(D.1)}$$

where

b = upper bound of the density function
a = lower bound of the density function
Γ = Gamma function

The expected value of the variable x is

$$E(x) = a + \frac{\alpha + 1}{\alpha + \beta + 2} (b-a) \qquad \text{(D.2)}$$

and the variance of the variable x is

$$V(x) = \frac{(b-a)^2 (\alpha + 1) (\beta + 1)}{(\alpha + \beta + 2)^2 (\alpha + \beta + 3)} \qquad \text{(D.3)}$$

When a, b, E(x) and V(x) are known, the constants, α and β, may be defined by the following

$$\alpha = \frac{\hat{x}^2}{\hat{v}} (1 - \hat{x}) - (1 + \hat{x}) \qquad \text{(D.4)}$$

$$\beta = \frac{\alpha + 1}{\hat{x}} - (\alpha + 2) \qquad \text{(D.5)}$$

where

$$\hat{x} = \frac{E(x) - a}{b - a} \tag{D.6}$$

$$\hat{v} = \frac{V(x)}{(b - a)^2} \tag{D.7}$$

APPENDIX E

PROBABILISTIC SLOPE STABILITY ANALYSIS BY THE POINT-ESTIMATES METHOD

The following program was developed to determine the probability of failure of a simple slope. Failure is defined to be a condition when there is any surface along which equilibrium cannot be maintained. Limit equilibrium is checked with the Taylor friction-circle method. The values of the soils friction angle, ϕ, cohesion intercept, c, and density, γ, are assumed to be symmetrically distributed. The mean value and the standard deviation of the strength factor of safety of the slope are obtained with a two-point estimate (Rosenblueth, 1975). A beta distribution is fitted to these statistical moments and the upper and lower bounds of the factor of safety. The probability of failure is computed by numerically integrating equation 3.13.

User Manual - Probabilistic Slope Stability Program

This program was developed on a Vax computer. All units are in English units, but any dimensionally homogeneous set of units will work. All input is unformatted.

Parameters Defining the Distribution of the Values of the Material Properties of the Slope

1. Input on one record:
   a) mean value of ϕ (degrees)
   b) the coefficient of variation of ϕ
   c) the lower bound of ϕ (degrees)
   d) the upper bound of ϕ (degrees)

2. Input on one record:
   a) the mean value of c (psf)
   b) the coefficient of variation of c
   c) the lower bound of c (psf)
   d) the upper bound of c (psf)

3. Input on one record:
   a) the mean value of γ (pcf)
   b) the coefficient of variation of γ
   c) the lower bound of γ (pcf)
   d) the upper bound of γ (pcf)

Parameters Defining the Geometry of the Slope

4. Input on one record:

a) the height of the slope (ft)

b) the slope angle (degrees)

Parameters Defining the Type of Limit Equilibrium Surface

5. Follow instructions 2-4 in the User Manual for the Taylor friction circle factor of safety which is found in Appendix C.

The program listing is included in the following pages.

```
      common/groovy/ nlimit,depthf
      common/hate/ numx,numy,toe,dangle,nsmax
      real mean(3),stddev(3),cv(3),bound(3,2),nlimit,nsmax
      integer toe
      data fsx,iout /1.0,1/
c
c
c
c     input material parameters
c      a)  mean value    b)  coefficient of variation
c      c)  lower and upper bounds of
c          1)  the friction angle
c          2)  the cohesion intercept
c          3)  the soil density
c
c
      do 5 i = 1,3
      read (5,*) mean(i),cv(i),bound(i,1),bound(i,2)
      stddev(i) = mean(i)*cv(i)
5     continue
      write (6,8)
8     format (15(/),t46,'soil properties',//,
     1       t20,'mean',t40,'coeff. of var.',t60,'lower bound',
     2       t80,'upper bound',//)
      write (6,12) mean(1),cv(1),bound(1,1),bound(1,2)
12    format (' phi',t18,f6.2,t42,f5.3,t62,f6.2,t82,f6.2)
      write (6,14) mean(2),cv(2),bound(2,1),bound(2,2)
14    format (' cohesion',t16,f8.1,t42,f5 3,t60,f8.1,t80,f8.1)
      write (6,16) mean(3),cv(3),bound(3,1),bound(3,2)
16    format (' density',t19,f5.1,t42,f5.3,t63,f5.1,t83,f5.1,5(/),
     1        t45,' slope geometry',//)
c
c     input geometry of the slope
c
      read (5,*) hight,slope
      write (6,17) hight,slope
17    format (t37,' slope hight = ',f5.1,/,t37,' slope =',f5.2)
      numx = int(slope) - 1
      numy = 89
      pi180 = acos(-1.0)/180.0
      dangle = pi180
      slope = slope*pi180
c
c      choose the type of failure surface
c
c     if the depth factor = 1.0                           set  toe = 1
c     if the depth factor > 1.0                           set  toe = 2
c     if the failure surface passes under the toe         set  toe = 3
c     if toe option = 2    input the depth factor,depthf
c     if toe option = 3    input the depth factor and the toe factor
c                          nlimit
c
c
```

```
      read (5,*) toe
c
      if (toe.eq.1) then
          write (6,18)
18        format (t37,' circles start have depth factor=1.0',//)
          numy = numx
          depthf = 1.0
          nlimit = 1.0
      endif
      if (toe.eq.2) then
          read (5,*) depthf
          write (6,19) depthf
19        format (t37,' circles start at the toe with depth factor=',f6.
     1            //)
      nlimit = 1.0
      endif
      if (toe.eq.3) then
          read(5,*) depthf,nlimit
          write (6,20) depthf,nlimit
20        format (t37,' circles extend beneath the toe',/,
     1            '          depth factor =',f6.1,/,
     2            '          toe factor   =',f6.1,//)
      endif
c
c     calculate the value of the central factor of safety
c
      write (6,21)
21    format (1h1,10(/),t20,'calculation of central fs')
      phmean = mean(1)*pi180
      call fsafty (slope,hight,phmean,mean(2),mean(3),fscntr)
      write (6,22) fscntr
22    format (/,t22,'central fs =',f6.3)
c
c     establish the lower bound of the capacity-demand functional
c
      phimin = bound(1,1)*pi180
      cmin = bound(2,1)
      denmax = bound (3,2)
      write (6,23)
23    format (5(/),t20,'calculation of min fs')
      phi1 = phimin/pi180
      write (6,25) phi1,cmin,denmax
25    format (/,t22,' phi = ',f6.2,/,
     1        t22,' cohesion = ',f8.1,/,
     2        t22,' density = ',f6.1)
      call fsafty (slope,hight,phimin,cmin,denmax,fsmin)
      write (6,26) fsmin
26    format (t22,'min fs =',f6.3)
      if (fsmin.ge.1.0) then
          print*,' probability of failure = 0'
          stop
      endif
c
```

```
c     establish the  upper bound of the capacity-demand
c     functional
c
      phimax = bound(1,2)*pi180
      cmax = bound(2,2)
      denmin = bound(3,1)
      write (6,27)
27    format (5(/),t20,'calculation of max fs')
      phi2 = phimax/pi180
      write (6,25) phi2,cmax,denmin
      call fsafty (slope,hight,phimax,cmax,denmin,fsmax)
      write (6,28) fsmax
28    format (t22,'max fs =',f6.3)
      if (fsmax.lt.1.0) then
          print*,' probability of failure = 1.0'
          stop
      endif
c
c     use the rosenbleuth approximation to approxmimate
c     the mean value and the variance of the capacity-
c     demand functional.
c     the following expressions for the point estimates
c     of the capacity-demand functional assume that the
c     coefficient of skewness of the phi,c,and gamma
c     parameters are all = 0 and that phi,c and gamma
c     are uncorrellated
c
      if (iout.eq.1) then
          write (6,40)
40        format (1h1,t30,'rosenbleuth point estimates')
          icount = 0
      endif
      sumfs = 0.0
      smfsqr = 0.0
      do 50 i = 1,2
            if (i.eq.1) phi = (mean(1) + stddev(1))*pi180
            if (i.eq.2) phi = (mean(1)  - stddev(1))*pi180
      do 50 j = 1,2
            if (j.eq.1) cohes = mean(2) + stddev(2)
            if (j.eq.2) cohes = mean(2) - stddev(2)
      do 50 k = 1,2
            if (k.eq.1) gamma = mean(3) + stddev(3)
            if (k.eq.2) gamma = mean(3) - stddev(3)
            if (iout.eq.1) then
                icount = icount + 1
                write (6,45) icount
45              format (5(/),t20,'point estimate',i3,/)
                phi3 = phi/pi180
                write (6,25) phi3,cohes,gamma
            endif
c
c      calculate the point estimator of the factor of safety
c
```

```
            call fsafty (slope,hight,phi,cohes,gamma,fs)
            write (6,46) fs
46          format (t20,'factor of safety=',f6.3)
            sumfs = sumfs + fs
            smfsqr = smfsqr + fs**2
50    continue
c
c
c
c     calculate the mean value of the point estimators of the fs
c
      fsmean = sumfs/8.0
c
c     calculate the variance of the point estimaters of the fs
c
      varfs = smfsqr/8.0 - fsmean**2
      sigma = sqrt(varfs)
c
c     calculate the probability that the fs will be < fsx=1.0,i.e.,
c     the probability that the slope can not maintain limit equilibrium
c
      call beta (fsmean,sigma,fsmin,fsmax,fsx,pfail)
      write (6,60) fscntr,fsmax,fsmin,v,pfail
60    format (1h1,20(/),t20,
     1          'properties of the capacity-demand functional',/,
     2          t30,'mean value =',f6.3,/,
     3          t30,'max  value =',f6.3,/,
     4          t30,'min  value =',f6.3,/,
     5          t30,'coef. of var. =',f6.3,/,
     6          t30,'prob. of failure=',f7.5)
      stop
      end
c
c
c
      subroutine fsafty (slope,hight,phi,cohes,gamma,fs)
      common/hate/ numx,numy,toe,dangle,nsmax
      common/lost/ coti,sinphi
      real nsmax
      integer toe,cycles
      data tol1,tol2,cycles,iout/1.0e-8,0.01,15,1/
c
c
      pi = acos(-1.0)
      if (iout.eq.1) write(6,10)
10    format (///,t10,'fs phi',t30,'fs cohes',t50,'stability number',//
20    format (t10,f5.3,t30,f5.3,t53,f7.4)
c
c
      if (abs(slope-pi/2.0).lt.tol1) then
          coti = 0.0
      else
          coti = 1.0/tan(slope)
```

```
      endif
      sinphi = sin(phi)
      cgamh = cohes/(gamma*hight)
      fsphi = 1.0
      fscohs = 0.0
      fc1old = 0.0
      fp1old = 0.0
c
c
      do 100 i = 1,cycles
c
c     estimate phi required for limit equilibrium
c
      phireq = atan(tan(phi)/fsphi)
      sinphi = sin(phireq)
c
      fc2old = fc1old
      fp2old = fp1old
      fc1old = fscohs
      fp1old = fsphi
c
      call phicir
c
c     back calculate the fs on the cohesion
c
      fscohs = cgamh/nsmax
      fratio = abs(fscohs-fsphi)/fscohs
c
c     compare the fs assumed on phi to the fs calculated on the cohesio
c
      if (fratio.lt.tol2) then
          if (iout.eq.1) write (6,20) fsphi,fscohs,nsmax
          fs = (fscohs + fsphi)/2.0
          return
      elseif (fratio.ge.tol2) then
          if (iout.eq.1) write (6,20) fsphi,fscohs,nsmax
          if (phi.lt.tol1) then
              fsphi = fscohs
          else
              diff1 = abs(fp2old-fsphi)
              diff2 = abs(fc2old-fscohs)
              if (diff1.lt.tol2.and.diff2.lt.tol2) then
                  fsphi = fscohs
              else
                  fsphi = (fsphi + fscohs)/2.0
              endif
          endif
      endif
      if (i.eq.cycles) then
          print*,' convergence not obtained'
          stop
      endif
100   continue
```

```
      end
c
c
c
      subroutine phicir
      common/hate/ numx,numy,toe,dangle,nsmax
      common/groovy/ nlimit,depthf
      common/lost/ coti,sinphi
      common/love/ en,x,y,cscx,cscy,cotx,coty,stbnum
      real nsmax,nlimit
      integer toe
c
c
c
c     y  is 1/2 the angle swept out by the circle in question
c
      y = 0.0
      nsmax = 0.0
      do 25 itery = 1,numy
      y = y + dangle
      coty = 1.0/tan(y)
      cscy = 1.0/sin(y)
      x = 0.0
      do 25 iterx = 1,numx
      x = x + dangle
      if (toe.eq.1.and.x.lt.y) go to 25
      cotx = 1.0/tan(x)
      cscx = 1.0/sin(x)
c
c     calculate the extent of the limit eqquilibrium surface beyond the
c     toe
c
      en = 0.5*(cotx - coty - coti + sinphi*cscx*cscy)
      if (toe.ne.3.and.en.gt.0.0) go to 25
      if (toe.eq.3.and.en.gt.nlimit) go to 25
c
c     calculate the depth factor
c
      d = 0.5*(cscx*cscy - cotx*coty + 1.0)
      if (toe.ne.1.and.d.gt.depthf.and.y.gt.x) go to 25
c
c     calculate the value of the stability number ffor the angles  x an
c
      call stabnm
c
c     choose the maximum value of the stability number
c
      nsmax = amax1(nsmax,stbnum)
25    continue
      return
      end
c
c
```

```
c
      subroutine stabnm
      common/love/ en,x,y,cscx,cscy,cotx,coty,stbnum
      common/lost/ coti,sinphi
c
c
c
c     use the friction circle method to determine the stability numbe
c     of a specified surface on the slope question
c     equations are from   taylor (1937)
c
      secx = 1.0/cos(x)
c
c     toe circle
c
      if (en.le.0.0) then
c
c     calculate   h/2d               eq. 9#
c
      param1 = (0.5*cscx**2*(y*cscy**2 - coty) + cotx
     1          - coti)/(1.0/3.0*(1.0 -2.0*coti**2) + coti
     2          *(cotx - coty) + cotx*coty)
c
c     calculate    cot(u)      .        eq. 10#
c
      param2 = param1*y*secx*cscx*cscy**2 - tan(x)
      u =  atan(1.0/param2)
c
c     calculate    sin(u-v)              eq. 11#
c
      param3 = param1*sin(u)*cscx*cscy*sinphi
      uv = asin(param3)
      v = u - uv
c
c     calculate the stability number    eq. 12#
c
      stbnum = (0.5*cscx**2*(y*cscy**2 - coty) + cotx -
     1          coti)/(2.0*cotx/tan(v) + 2.0)
c
c     slip surface beneath the toe of the slope
c
      elseif (en.gt.0.0) then
c
c     eq. 14#
c
      param1 = (0.5*cscx**2*(y*cscy**2 - coty) + cotx  -
     1          coti - 2.0*en)/(1.0/3.0*(1.0 - 2.0*coti
     2          **2) + coti*(cotx - coty) + cotx*coty +
     3          2.0*en**2 - 2.0*en*sinphi*cscx*cscy)
c
c     eq. 15#
c
      param2 = param1*y*secx*cscx*cscy**2 - tan(X)
```

```
      u = atan(1.0/param2)
c
c     eq. 16#
c
      param3 = param1*sin(u)*cscx*cscy*sinphi
      uv = asin(param3)
      v = u - uv
c
c     stability number     eq. 17#
c
      stbnum = (0.5*cscx**2*(y*cscy**2 - coty) + cotx  -
     1          coti - 2.0*en)/(2.0*cotx/tan(v) + 2.0)
      endif
      return
      end
c
c
c
      subroutine beta (xbar,sx,xmin,xmax,x,pf)
      real const(5)
c
c     subroutine to find the probability of given points
c     by using a beta distribution fitted knowing the
c     mean, standard deviation and range of all data.
c     formulas from  'mechanics of particulate media' by m.e.harr
c
      const(1) = xmin
      const(2) = xmax
c
c     calculate  x-tilda   pg. 496  eq. c-30
c
      xt=(xbar-const(1))/(const(2)-const(1))
c
c     calculate  variance-tilda          eq. c-30
c
      vt=(sx/(const(2)-const(1)))**2
c
c     calculate   alpha         eq. c-31a
c
      const(3)=xt**2*(1.0-xt)/vt-(1.0+xt)
c
c     calculate  beta            eq. c-31b
c
      const(4)=(const(3)+1.0)/xt-(const(3)+2.0)
      const(5) = 1.0
      cal=qudrtr(const(1),const(2),const)
      const(5)=1.0/cal
      pf = qudrtr(const(1),x,const)
      return
      end
c
c
c
```

```
      function qudrtr(a,b,coeff)
      real xg(8),wg(8),coeff(5)
      data xg,wg
     1     /0.0950125098,0.2816035507,0.4580167776,0.6178762444,
     2     0.7554044083,0.8656312023,0.9445750230,0.9894009349,
     3     0.1894506104,0.1826034150,0.1691565193,0.1495959888,
     4     0.1246289712,0.0951585116,0.0622535239,0.0271524594/
c
c
c     integrates the function betaf between the limits a and b
c     by sixteen point symmetric gaussian quadrature.
c     coeff is a real array of coefficients
c     for use in the function betaf, assumed independent of the
c     variable over which the integration is taking place.
c
c
      sum=0.0
      amb=(b-a)*0.5
      apb=(b+a)*0.5
      do 1 i=1,8
           xp=apb+amb*xg(i)
           xm=apb-xg(i)*amb
           sum=sum+wg(i)*(betaf(xp,coeff)+betaf(xm,coeff))
1     continue
      sum=sum*amb
      qudrtr=sum
      return
      end
c
c
c
      function betaf(x,coeff)
      real coeff(5),x
c
c     computes beta function
c          coeff(1)=a
c          coeff(2)=b
c          coeff(3)=alpha
c          coeff(4)=beta
c          coeff(5)=normalizing constant at the point x,
c                    where  a < x < b
c
c
c     eq. c-27a
c
      betaf = coeff(5)*(x-coeff(1))**coeff(3)*(coeff(2)-x)**coeff(4)
      return
      end
```

APPENDIX F

STRESSES CAUSED BY CONSTRUCTION OF AN EMBANKMENT

The following expressions may be used to calculate the stress changes caused by a semi-embankment loading in a semi-infinite, weightless, linear elastic half-space (Jurgenson, 1940):

$$\Delta\sigma_z = \frac{P}{\pi}\left(\beta + \frac{x}{a}\alpha - \frac{z}{r_2^2}(x-b)\right) \tag{F.1}$$

$$\Delta\sigma_x = \frac{P}{\pi}\left(\beta + \frac{x}{a}\alpha + \frac{z(x-b)}{r_2^2} + \frac{2z}{a}\ln\left(\frac{r_1}{r_o}\right)\right) \tag{F.2}$$

$$\Delta\tau_{xz} = \frac{P}{\pi}\left(\frac{z}{a}\alpha - \frac{z}{r_2^2}\right) \tag{F.3}$$

where

$P = \gamma \cdot H$

γ = embankment soils density

H = embankment height

β, α, a, b, r_o, r_1, r_2 are illustrated in Figure F.1.

Equations F.1 to F.3 assume:

1) The ground surface is horizontal.

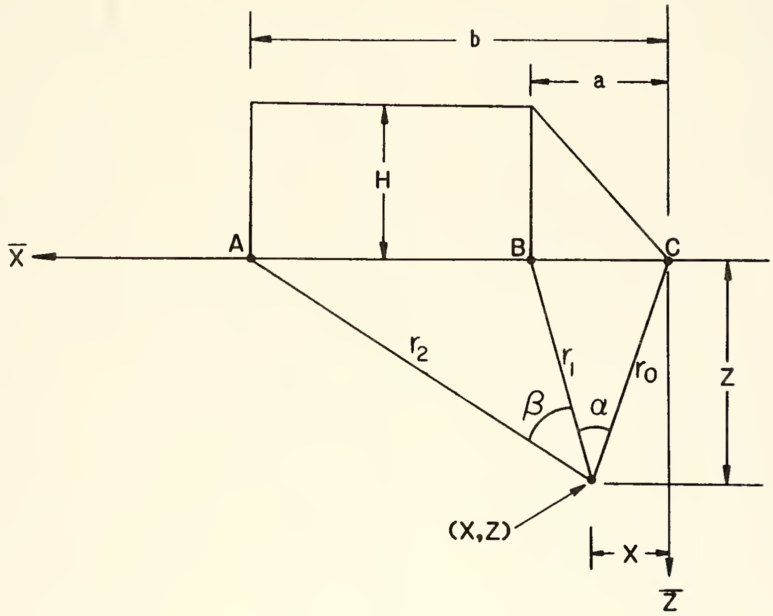

FIGURE F.1

INFINITELY LONG, PERFECTLY FLEXIBLE, SEMI–EMBANKMENT LOADING OVER A SEMI–INFINITE LINEAR ELASTIC MEDIUM

2) The embankment is perfectly flexible. This is equivalent to saying that the embankment - halfspace interface is frictionless.

3) Strains are infinitesimal.

4) The embankment is infinite in its linear direction.

The stress changes for an entire embankment may be obtained by summing the stress contributions of each half of the embankment with equations F.1 to F.3. It should be noted that the stresses cannot be calculated at positions A, B, or C in Figure F.1.

Given $\Delta\sigma_x$, $\Delta\sigma_z$, and $\Delta\tau_{xz}$, the principal stress changes, $\Delta\sigma_1$ and $\Delta\sigma_3$ are calculated. The stresses in the linear direction depend on the values of elastic constants. Since the problem is one of plane strain:

$$\Delta\sigma_2 = \nu \, (\Delta\sigma_1 + \Delta\sigma_3) \tag{F.4}$$

where ν = Poisson's ratio.

The value of Poisson's ratio does not affect the values of $\Delta\sigma_x$, $\Delta\sigma_z$, $\Delta\tau_{xz}$, $\Delta\sigma_1$, or $\Delta\sigma_3$.

Instructions for a computer program that facilitates computation of equations F.1 to F.4 are included on the following pages.

User Manual - Stress Under an Embankment

The following program contained hereafter was developed on a CDC 6000 series computer using 77 FORTRAN. Input is intended to be in English units, but other dimensionally homogeneous units may be employed. All input is unformatted.

1. Read in on one record:
   a) the side slope of the right hand side of the embankment (degrees)
   b) the side slope of the left hand side of the embankment (degrees)
   c) the embankment height (ft)
   d) the crest to crest width (ft)
   e) the density of the embankment soil (pcf)

2. Read in on one record:
   a) the Poisson's ratio of the soil

3. Read in on one record:
   a) the number of (X,Z) coordinate pairs at which stresses are desired (integer)

4. Read in on one record:
   a) the X coordinate of the point at which stresses are desired (ft)
   b) the Z coordinate of the point at which stresses are desired (ft)

Repeat for each coordinate pair.

X is measured from the center line of the crest of the embankment (see Figure F.2).

Z is measured downwards from the ground surface (see Figure F.2).

The program listing is included in the following pages.

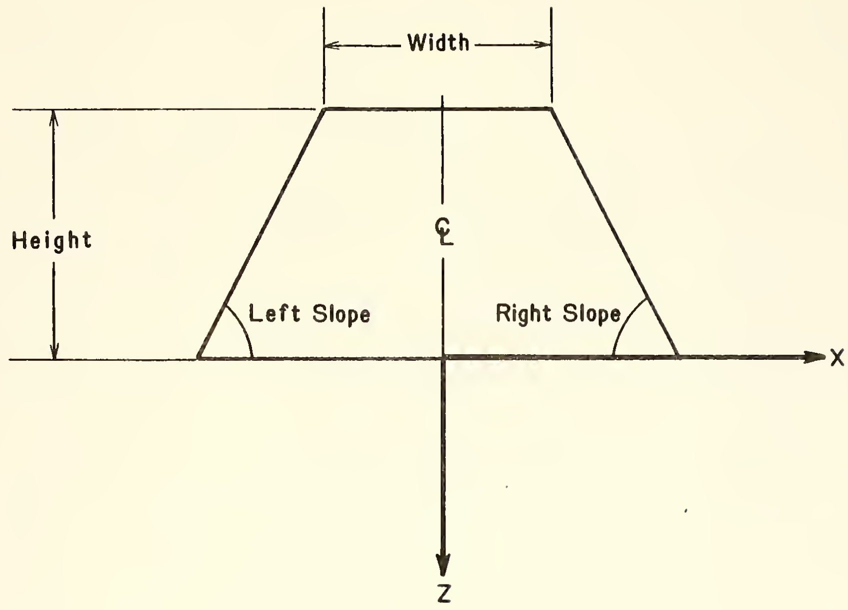

FIGURE F.2 EMBANKMENT GEOMETRY FOR STRESS PROGRAM

```
      program main (input,output,tape5=input,tape6=output)
c
      common x,z,hembnk,width,densty,sigmaz,sigmax,sigmxz,sigma1,
     1        sigma2,sigma3,theta,mu,slope1,slope2,pi
c
      real mu
c
c
c
c
c           this program calculates the values of stress change caused
c           by an embankment on a linear elastic half-space.
c
c
c
c
c           read in
c           1) the righthand side side-slope of the embankment in
c              degrees.
c           2) the lefthand side side-sloe of the embankment in
c              degrees.
c           3) the embankment hight
c           4) the crest width
c           5) the embankment's density
c
      read (5,*) slope1,slope2,hembnk,width,densty
      write (6,5) slope1,slope2,hembnk,width,densty
5     format ('1',5(/),t50,'right side-slope = ',f8.2,' degrees.',/,
     1          t50,'left side-slope = ',f8.2,' degrees.',/,
     2          t50,'embankment hight = ',f10.2,' feet.',/,
     3          t50,'crest width = ',f6.2,' feet.',/,
     4          t50,'embankment density = ',f7.2,' lb./ft**3')
c
      pi = acos(-1.0)
c
c           read in the value of poissons' ratio for the elastic
c           half-space.
c
      read (5,*) mu
      write (6,3) mu
3     format (//,t50,'mu = ',f4.2)
c
c           read in the number of coordinate pairs that the stress
c           change is desired at.
c
      read (5,*) npairs
      write (6,10)
10    format ('1',t8,'x',t22,'z',t38,'sigma z ',t52,'sigma x',t66,
     1          'sigma xz',t81,'sigma1',t95,'sigma2',t109,'sigma3',
     2          t124,'theta',//)
c
c
c           calculation loop   input an (x,z) coordinate pair
```

```
c            for each loop
c
      do 100 i =1,npairs
            read (5,*) x,z
            call stress
            write (6,20) x,z,sigmaz,sigmax,sigmxz,sigma1,sigma2,sigma3
     1                     theta
20          format (2(f9.1,6x),6(5x,f9.1),5x,f9.2)
100   continue
      end
      subroutine stress
      common x,z,hembnk,width,densty,sigmaz,sigmax,sigmxz,sigma1,
     1        sigma2,sigma3,theta,mu,slope1,slope2,pi
c
      real mu
c
c
      p = densty * hembnk
c
c
c          contribution to the stresses from the right half of the
c          embankment loading
c
c
      sloper = slope1 * pi/180.0
      ar = hembnk/tan(sloper)
      br = ar + width/2.0
      x1 = br - x
      r2 = sqrt((width/2.0 + ar - x1)**2 + z**2)
      r1 = sqrt((x1 - ar)**2 + z**2)
      r0 = sqrt(x1**2 + z**2)
      betar = acos((r1**2 + r2**2 - (width/2.0)**2)/(2.0*r1*r2))
      gammar = acos((r1**2 + r0**2 - ar**2)/(2.0*r1*r0))
      ciflr = betar + x1/ar * gammar - z*(x1-br)/r2**2
      rightx = betar + x1/ar*gammar + z*(x1-br)/r2**2 + 2.0*z/ar*alog
     1        (r1/r0)
      rihtxz = z/ar*gammar - z**2/r2**2
c
c
c          contribution to the stresses from the left side.
c
c
      slopel = slope2*pi/180.0
      al = hembnk/tan(slopel)
      bl = al + width/2.0
      x2 = bl + x
      r2 = sqrt((width/2.0 + al - x2)**2 + z**2)
      r1 = sqrt((x2 - al)**2 + z**2)
      r0 = sqrt(x2**2 + z**2)
      betal = acos((r1**2 +r2**2 - (width/2.0)**2)/(2.0*r1*r2))
      gammal = acos((r1**2 + r0**2 - al**2)/(2.0*r1*r0))
      cifl1 = betal + x2*gammal/al - z*(x2-bl)/r2**2
      leftx = betal + x2/al*gammal + z/r2**2*(x2 - bl) + 2.0*z/al*alog
```

```
     1          (r1/r0)
      leftxz = z/a1*gamma1 - z**2/r2**2
      sigmaz = p/pi*(ciflr + cifl1)
c
c
c
c
      sigmax = p/pi*(rightx + leftx)
      sigmxz = p/pi*(rihtxz + leftxz)
      descr = sqrt ((sigmax - sigmaz)**2/4.0 + sigmxz**2)
      center = (sigmax + sigmaz)/2.0
      sigma1 = center + descr
      sigma3 = center - descr
      sigma2 = mu*(sigma1 + sigma3)
      theta = (0.5*atan(-2.0*sigmxz/(sigmaz - sigmax)))*180.0/pi
c
c
      return
      end
```

APPENDIX G

MAGNITUDE OF CONSOLIDATION SETTLEMENT

The program contained hereafter computes the magnitude of consolidation settlement of compressible soil layers caused by construction of an embankment. Up to ten layers are permitted. Each layer may be automatically divided into any number of strata. The preconsolidation pressure profile is input as shown in Figure 4.2b. The void ratio is automatically corrected for depth with equations 4.6a to 4.6h. Settlement is computed with equations 4.2 to 4.5.

The program was developed on a CDC 6000 series computer using 77 FORTRAN.

User Manual - Magnitude of Consolidation Settlement Program

Input is intended to be in English units, but other dimensionally homogeneous units may be employed.

All input is unformatted.

Specify the Number and Type of Layers in the Soil Profile:

1. Read in on one record:

   a) the total number of soil layers (integer)

   b) the number of compressible soil layers (integer)

   c) the number of layers with a portion above the ground water table (integer)

   d) the number of layers with a portion below the ground water table (integer)

Specify the Compressibility Models for Each Layer:

2. For each soil layer, read in on one record:

   a) the layer number (integer)

   b) the thickness of the layer (feet)

   c) An identifier variable that indicates if the soil layer is compressible (integer).
      If the soil is not considered to be compressible, input '0'
      If the soil is considered to be compressible, input '1'.

   d) If the soil is considered to be compressible, read

in the following variables on a separate record prior to inputting (a)-(c) for the subsequent soil layer.

i) An option variable that is used to control the manner by which e_o and σ'_p are assumed to vary with depth in the soil layer (integer)
If one value of e_o and σ'_p are used to represent the entire soil layer, input '1'.
If e_o and σ'_p are specified at three depths within the soil layer, input '2'.

ii) An option variable that specifies if the soil is to be treated as normally consolidated or underconsolidated at depths beneath which the calculated value of σ'_v is greater than the value of σ'_p (integer)
If the soil layer is to be treated like normally consolidated soil at depths where σ'_v is greater than σ'_p, input '1'.
If the soil layer is to be treated as an underconsolidated soil at depths where σ'_v is greater than the value of σ'_p, input '2'.

Specify the Ground Water Table:

3. Read in on one record:

a) the depth of the groundwater table beneath the ground surface (feet).

The depth is measured downwards from the ground surface (see Figure F.2).

Specify the Saturated Density of the Soil Layers:

4. For each layer with a saturated zone, read in on one record:

   a) the layer number (integer)

   b) the saturated density of the soil layer (pcf)

Specify the Unsaturated Density of the Soil Layers:

5. For each layer with an unsaturated zone, read in on record:

   a) the layer number (integer)

   b) the density of the soil layer (pcf)

Specify the Compressibility Parameters of Each Layer:

6. For each compressible layer, read in on one record:

   a) the layer number (integer)

   If one value of e_o and σ'_p represent the entire layer read in on a separate record:

   b) the initial void ratio

   c) the preconsolidation pressure (psf)

   d) the compression index

   e) the recompression index

   f) the depth beneath the ground surface from which the tested sample was taken (ft)

If the soil may be considered to be underconsolidated, input on a separate record:

b) the compression index

c) the recompression index

Also, if the soil is underconsolidated, input the following items on the subsequent record:

d) e_o at depth 1#

e) σ'_p at depth 1# (psf)

f) Depth 1# (ft)

g) e_o at depth 2#

h) σ'_p at depth 2# (psf)

i) Depth 2# (ft)

j) e_o at depth 3#

k) σ'_p at depth 3# (psf)

l) Depth 3# (ft)

Specify the Embankment Load:

7. Read in on one record:

a) the height of the embankment (feet)

b) the sideslope angle of the embankment (degrees)

c) the width of the embankment crest (feet)

d) the density of the embankment soil (pcf)

Specify the Lateral Limits of the Settlement Calculations:

8. Read in on one record:

   a) the leftmost bound for which settlements will be calculated (feet)

   b) the rightmost bound for which settlements will be calculated (feet)

   c) the number of evenly spaced points along the embankments cross section at which settlement will be calculated (integer)

   Coordinates for these bounds are measured (+) and (−) to the right and left of the embankment centerline respectively.

   If the settlement of only one profile is desired, the program calculates settlement at the left boundary.

Specify the Number of Strata Compressible Layers are Divided into:

9. For each compressible layer, read in on one record:

   a) the layer number (integer)

   b) the number of strata into which the layer will be divided into for purposes of analysis (integer)

The program listing is contained in the following pages.

```
c
      parameter (nprfil = 11, nlyrs = 10, nlyrs1 = 11)
c
      integer comprs(nlyrs),strata(nlyrs),ocr(nlyrs),iprfil(nlyrs),
     1        iout
c
       real  thick(nlyrs1),densat(nlyrs),denmst(nlyrs),
     1       cc(nlyrs),total(nprfil),settle(nprfil,nlyrs),
     2       rc(nlyrs),x11(nprfil),eo1(nlyrs),eo2(nlyrs),
     3       eo3(nlyrs),pc1(nlyrs),pc2(nlyrs),pc3(nlyrs),zsamp1(nlyrs),
     4       zsamp2(nlyrs),zsamp3(nlyrs)
c
c
      data gammaw/62.4/,tol/.001/
c
c
c           program assumes e-logp behaviour only in saturated zone.
c
c
c           units of length - (feet)
c           units of force  - (pounds)
c           units of pressure - (psf)
c
c
      write(6,1)
      write(6,2)
c
c
      pi = acos(-1.0)
c
c           read in:
c           1) total # of soil layers
c           2) # of compressible soil layers
c           3) # of layers with portions above the ground water table.
c           4) # of layers with portions below the ground water table.
c
c
c
      read (5,*) nlayer,nclayr,nabove,nbelow
c
c
c
      do 4 i=1,nlayer
            ocr(i) = 0
            iprfil(i) = 0
4     continue
c
c           read in geometry of problem
c
c
c           for each layer read in:
c           1) layer #
c           2) the thickness of the i'th layer.
```

```
c          3) if the layer is compressible.
c             if,yes then input  '1'
c             if,no  then input  '0'
c          4) if the layer is compressible, read in on a seperate
c             card:
c             iprfil
c                   if the variation of pc and eo is defined by
c                   two line segments,  set iprfil = 2
c                   if only one value of pc and eo are specified
c                   for the layer, set iprfil = 1
c               ocr
c                   if pc is to be automatically corrected to
c                   the overburden pressure when pc<overburden
c                   pressure,  set ocr = 1
c                   if pc is to be considered underconsolidated
c                   when pc < overburden pressure,
c                              set ocr = 2
c
c               ocr = 1      if the soil is normally consolidated
c                            at depths beneath which
c                           the overburden pressure is greater than the
c                           value of the preconsolidation pressure that
c                           was input
c               ocr = 2     if the preconsolidtion pressure is constant
c                           with depth.
c
c
c
      do 5 i=1,nlayer
c
c
c
            read (5,*) i1,thick(i1),comprs(i1)
c
c
c
            if (comprs(i1).eq.1) read (5,*) iprfil(i1),ocr(i1)
c
c
5     continue
c
      do 10 i=1,nlayer
            write (6,6) i,thick(i)
                  if (comprs(i).eq.1) then
                    if (ocr(i).eq.1.or.iprfil(i).eq.2) then
                      write (6,7)
                    elseif (ocr(i).eq.2) then
                      write (6,8)
                    endif
                  elseif (comprs(i).eq.0) then
                          write (6,9)
                  endif
10    continue
```

```
c
c
c
c
c
c
c          read in elevation of ground water surface
c
c
      read (5,*) zwater
      write (6,11) zwater
c
c
c
      write (6,20)
c
c
c           initialize moist and saturated densities of each layer.
c
c
      do 24 i=1,nlayer
            denmst(i) = 0.0
            densat(i) = 0.0
24    continue
c
c
c           for each layer with a saturated zone, read in:
c           1) layer #
c           2) saturated density
c
c
c
c
c
      do 25 i=1,nbelow
c
c
            read (5,*) i1,densat(i1)
c
c
25    continue
c
c
c           for each layer with an unsaturated zone, read in:
c           1) layer #
c           2) moist density
c
c
      do 30 i=1,nabove
c
c
            read (5,*) i1,denmst(i1)
c
```

```
c
30    continue
c
c
c               read in compressibility properties
c
c
c
      do 40 i = 1,nclayr
c
c
c           input the number of the compressible layer
c
c
      read (5,*) i1
c
c
c
         if (iprfil(i1).eq.1) then
c           for each compressible layer, read in:
c           1) initial void ratio
c           2) preconsolidation pressure (psf)
c           3) compression index (log 10)
c           4) recompression index (log 10)
c           5) sample depth (feet)
c
c
c
            read (5,*) eo1(i1),pc1(i1),cc(i1),rc(i1),zsamp1(i1)
c
c
c
          elseif (iprfil(i1).eq.2) then
c
c
            read (5,*) cc(i1),rc(i1)
            read (5,*) eo1(i1),pc1(i1),zsamp1(i1),eo2(i1),pc2(i1),
     1                 zsamp2(i1),eo3(i1),pc3(i1),zsamp3(i1)
c
c
          endif
40    continue
c
c
c           output material parameters
c
c
      write (6,50)
      do 60 i=1,nlayer
            if (abs(denmst(i)).lt.tol) then
                write (6,54) i,densat(i)
            elseif (abs(densat(i)).lt.tol) then
                write (6,56) i,denmst(i)
```

```
          elseif(densat(i).gt.0.0.and.denmst(i).gt.0.0) then
                 write (6,57) i,densat(i),denmst(i)
          endif
          if (comprs(i).eq.1) then
            if (iprfil(i).eq.1) then
              write (6,58) eo1(i),pc1(i),cc(i),rc(i),zsamp1(i)
         elseif (iprfil(i).eq.2) then
              write (6,61) eo1(i),pc1(i),cc(i),rc(i),zsamp1(i),
     1                     eo2(i),pc2(i),zsamp2(i),eo3(i),pc3(i),
     2                     zsamp3(i)
          endif
        elseif (comprs(i).eq.0) then
               write (6,59)
      endif
60    continue
      write (6,67)
c
c
c
c
c         read in dimensions of embankment load:
c
c         1) embankment hight
c         2) sideslope of embankment (degrees)
c         3) crest width
c         4) density of embankment material during period of
c            consolidation.
c
c
c
      read (5,*) hembnk,slope1,width,densty
      write (6,68) hembnk,slope1,width,densty
c
c
c
c
c        read in:
c        1) the lateral limits between which settlement is calculated
c        2) the # of points between the bounds for which
c           consolidation is to be calculated.
c
c
      read (5,*) boundl,boundr,nntrv1
c
c
      nintrv = nntrv1 - 1
      write (6,72) nntrv1,boundl,boundr
      if (nntrv1.gt.1)then
                  dx = (boundr - boundl)/float(nintrv)
      else if (nntrv1.eq.1) then
                 dx = 0.0
      endif
      write (6,74)
```

```
c
c
c
c         for each compressible layer, read in:
c         1) layer #
c         2) the # of strata the layer will be divided into for
c            purposes of analysis
c
c
      do 80 i=1,nclayr
c
c
              read (5,*) i1,strata(i1)
c
c
              write (6,75) i1,strata(i1)
80    continue
c
c
c
c          if strains are to be output for each strata
c                     set  iout = 1
c          otherwise  set  iout = 0
c
      read (5,*) iout
c
c
c
      x1 = boundl - dx
c
      if (iout.eq.1) write (6,85)
c          this loop sums settlement from each layer  beneath
c          each point for which settlement is desired.
c
c
c
c
      do 200 np= 1,nntrvl
      ztop =0.0
      prtop = 0.0
      x1 = x1 + dx
      if (iout.eq.1) write (6,90) x1
      x = abs (x1)
      total(np) = 0.0
c
c
c          this loop calculattes settlement of each layer
c          beneath the np'th layer.
      do 190 i=1,nlayer
      if (iout.eq.1) write (6,91) i,thick(i)
      settle (np,i) = 0.0
c
      zbottm = ztop + thick(i)
```

```
c
c            calculate the pressure at the bottom of the
c            i'th layer.
c
          if (zwater.gt.ztop.and.zwater.lt.zbottm) then
              prbotm = prtop + (zwater-ztop)*denmst(i) + (zbottm-zwater)
         1              *(densat(i)-gammaw)
          else if (zwater.le.ztop) then
                   prbotm = prtop + (zbottm-ztop)*(densat(i)-gammaw)
          else if (zwater.ge.zbottm) then
                   prbotm = prtop + (zbottm-ztop)*denmst(i)
          endif
c
          if (comprs(i).eq.1) then
              dz = (zbottm-ztop)/float(strata(i))
c
            if (iprfil(i).eq.1) then
c
c            calculate the sample pressure
c
          if (zwater.le.ztop) then
              psampl = prtop + (densat(i) - gammaw)*(zsamp1(i)-ztop)
          else if (zwater.gt.ztop.and.zwater.lt.zsamp1(i)) then
          psampl = prtop + denmst(i)*(zwater-ztop) + (denmst(i)-gammaw)*
         1         (zsamp1(i)-zwater)
          endif
          endif
c
          nstrta = strata(i)
c
c
c
c            this loop divides each compressible layer into
c            distinct strata for purposes of calculating
c            settlements.
c
          do 100 n=1,nstrta
             z = ztop + float(2*n-1)/2.0*dz
          if (iout.eq.1) write (6,92) n,z
c
c
c            calculate the insitu pressure at the midpoint
c            of the n'th strata.
c
          if (zwater.le.ztop) then
              pr1 = prtop + (densat(i)-gammaw)*(z-ztop)
          else if (zwater.gt.ztop.and.zwater.lt.z) then
                   pr1 = prtop + (zwater-ztop)*denmst(i) + (z-zwater)*
         1               (densat(i)-gammaw)
          endif
c
c
          if (z.le.zwater) then
```

```
c             set the settlement of the strata = 0  above the ground
c             water table.
c
          sqeeze = 0.0
c
c
        elseif (zwater.lt.z) then
c
c
c             calculate eo at the center of each strata
c             beneath th water table.
c
c
        if (iprfil(i).eq.1) then
c
        if (ocr(i).eq.2) then
             precon = pc1(i)
        elseif (ocr(i).eq.1.and.pr1.gt.pc1(i)) then
          precon =pr1
        elseif (ocr(i).eq.1.and.pr1.le.pc1(i)) then
              precon = pc1(i)
        endif
c
c             center of strata above the samples elevation
c
        if (z.lt.zsampl(i)) then
c
          if (psampl.le.precon) then
c
c                       case  i
c             the soil is over consolidated above the sample
               estrat = eo1(i) + rc(i)*alog10(psampl/pr1)
c
c
          elseif (psampl.gt.precon) then
               if (ocr(i).eq.2) then
            if(pr1.gt.precon) then
c
c                       case  ii
c             the soil is underconsolidated and therefore will
c             have a constant void ratio at pressures above
c             the preconsolidation pressure.
                       estrat = eo1(i)
c
c
c
            else if (pr1.le.precon) then
c
c                       case  iii
c             the soilis underconsolidated at pressures above
c             the p'c and overconsolidated at pressures be-
c             low the p'c
                      estrat = eo1(i) + rc(i)*alog10(precon/pr1)
```

```
c
c
c
        endif
          elseif (ocr(i).eq.1) then
c
c                   case  iv
c         the soil is normally consolidated at pressures
c         above the p'c and over-consolidated at pressures
c         beneath the p'c
                estrat = eo1(i) + cc(i)*alog10(psampl/precon)
     1                        +  rc(i)*alog10(precon/pr1)
c
c
c
          endif
       endif
c
c         center of the strata below the samples elevation
c
     else if (z.ge.zsampl(i)) then
        if (psampl.lt.precon) then
          if (pr1.le.precon) then
c
c                   case  v
c         the soil is over-consolidated
              estrat = eo1(i) - rc(i)*alog10(pr1/psampl)
c
c
c
          else if (pr1.gt.precon) then
         if (ocr(i).eq.1) then
c
c                   case  vi  a)
c         the soil is over-consolidated below the p'c and normally
c         consolidated above the p'c
                  estrat = eo1(i) - rc(i)*alog10(precon/psampl) -
c
c
     1                          cc(i)*alog10(pr1/precon)
         elseif (ocr(i).eq.2) then
c
c
c                   case v1 b)
c         the soil over-consolidated below the p'c and overconsolidate
c         above the p'c
c
c
         estrat = eo1(i) - rc(i)*alog10(precon/psampl)
       endif
c
          endif
c
```

```
          else if (psampl.ge.precon) then
c
c                      case  vii
c            the soil is underconsolidated
c
                   estrat = eo1(i)
          endif
c
        endif
        elseif (iprfil(i).eq.2) then
              if (z.le.zsamp2(i)) then
                  precon = pc2(i) + (pc2(i) - pc1(i))/(zsamp2(i) -
     1                     zsamp1(i))*(z - zsamp2(i))
                  estrat = eo2(i) + (eo2(i) - eo1(i))/(zsamp2(i) -
     1                     zsamp1(i))*(z - zsamp2(i))
              elseif (z.gt.zsamp2(i)) then
                  precon = pc2(i) + (pc3(i) - pc2(i))/(zsamp3(i) -
     1.                    zsamp2(i))*(z - zsamp2(i))
                  estrat = eo2(i) + (eo3(i) - eo2(i))/(zsamp3(i) -
     1                     zsamp2(i))*(z - zsamp2(i))
              endif
             if (precon.lt.pr1.and.ocr(i).eq.1) precon = pr1
        endif
        call stress (x,z,hembnk,slope1,width,densty,sigmaz,pi)
        sumsig = sigmaz + pr1
c
c
c            calculate the compression of the strata
c
c
        if (sumsig.le.precon) then
            sqeeze = dz/(1.0 + estrat)*rc(i)*alog10(sumsig/pr1)
        else if (sumsig.gt.precon) then
           if (pr1.le.precon) then
                 sqeeze = (dz/(1.0 + estrat))*(rc(i)*alog10(precon/pr1) +
     1                    cc(i)*alog10(sumsig/precon))
           elseif (pr1.gt.precon) then
                 if (ocr(i).eq.1) then
                 sqeeze = dz/(1.0 + estrat)*(cc(i)*alog10(sumsig/pr1))
                 elseif (ocr(i).eq.2) then
                 sqeeze = dz/(1.0 + estrat)*(cc(i)*alog10
     1                   (sumsig/precon))
                 endif
           endif
        endif
c
        endif
c
c
        if (iout.eq.1) then
            strain = sqeeze/dz*100.0
            write (6,99) strain
        endif
```

```
c           sum the settlements of each strata in the layer.
c
      settle (np,i) = settle (np,i) + sqeeze
c
100   continue
c
c
      endif
c
c
c           reset the elevation and pressure at the top
c           of the layer beneath the current layer.
c
      ztop = zbottm
      prtop = prbotm
c
c           sum the settlements of each layer beneath the
c           the point at which the settlement is desired.
c
      total(np) = total(np) + settle(np,i)
190   continue
200   continue
c
c          output statements
c
c
      write (6,210)
      x11(1) = boundl
c
      if (nntrvl.ge.2) then
      do 300 np = 2,nntrvl
                  x11(np) = x11(np - 1) + dx
300   continue
      endif
c
      write (6,250) (x11(np),np=1,nntrvl)
c
      write (6,305)
c
      do 400 i=1,nlayer
         if (comprs(i).eq.1) then
             write (6,310) i,(settle(np,i),np=1,nntrvl)
         endif
400   continue
c
      write (6,410) (total(np),np = 1,nntrvl)
c
c
c              formats
c
c
1     format('1',25(/),t30,66('*'),/,
     1       t30,'*',t95,'*',/,t30,'*',t95,'*',/,t30,'*',t95,'*',/,
```

```
     2         t30,'*',t55,'program elogp',t95,'*',/,
     3         t30,'*',t40,'this program calculates the settlement beneat
     4         ',t95,'*',/,
     5         t30,'*',t40,'an infinitely long embankment load due to the
     6         ,t95,'*',/,
     7         t30,'*',t44,'compression of saturated clay layers.',t95,
     8         '*',/,
     9         t30,'*',t95,'*',/,t30,'*',t95,'*',/,t30,'*',t95,'*',/,t30,
     1         66('*'))
2     format('1',5(/),t58,'problem geometry',5(/),
     1         t38,'layer',15x,'thickness (feet)',8x,'compressible ?',
     2         //)
6     format(t39,i2,t60,f6.1)
7     format('+',t88,'yes')
8     format ('+',t82,'underconsolidated')
9     format('+',t88,'nO')
20    format(5(/),t52,'properties of the soil profile',/////)
50    format (t2,'layer',t10,'sat. density',
     1         5x,'moist density',7x,'void ratio',
     2         5x,'preconsolidation',5x,'compression',1x,'recompression'
     3         ,5x,'sample depth',/,
     4         t49,'initial',t64,'pressure',t86,'index',t100,
     5         'index',/,t13,'(pcf)',t31,'(pcf)',t67,'(psf)',t116,
     6         '(feet)',5(/))
54    format(t3,i2,t13,f5.1,t31,'*****')
56    format(t3,i2,t13,'*****',t31,f5.1)
57    format(t3,i2,t13,f5.1,t31,f5.1)
58    format('+',t50,f5.3,t66,f8.1,t86,f5.3,t100,f5.3,t117,f6.1)
59    format('+',t50,'*****',t66,'********',t86,'*****',t100,'*****',
     1         t117,'*****')
11    format(3(/),35x,'the phreatic surface lies',f8.1,' feet beneath
     1the ground surface.')
61    format ('+',t50,f5.3,t66,f8.1,t86,f5.3,t102,f5.3,
     1         t117,f6.1,/,t50,f5.3,t66,f8.1,t117,f6.1,/,
     2         t50,f5.3,t66,f8.1,t117,f6.1)
67    format('1',5(/),t50,'dimensions of embankment load',//)
69    format(t50,'embankment hight = ',f4.1,' feet.',/,
     1         t50,'sideslope = ',f4.1,' degrees.',/,
     2         t50,'crest width = ',f5.1,' feet.',/,
     3         t50,'density of the applied load = ',f5.1,' pcf.')
72    format(3(/),t18,'settlement will be calculated at ',i2,
     1         ' equally spaced positions between x= ',f5.1,' and x= ',
     2         f5.1,' feet.',3(/))
74    format(t45,'layer #',15x,'no. of divisions',//)
75    format(t47,i2,t74,i2)
85    format ('1',t25,'x  (feet)',t40,'layer',t50,'strata',
     1         t66,'depth  (feet)',t84,'thickness (feet)',
     2         t105,'strain (percent)',3(/))
210   format('1',t50,'settlement (feet)',////,t55,'x (feet)',//)
90    format (t26,f6.1)
91    format ('+',t41,i2,t88,f8.1)
92    format ('+',t52,i2,t63,f7.1)
99    format ('+',t110,f7.2)
```

```
250    format(t25,11(f5.1,5x))
305    format(//,t11,'layer')
310    format(t12,i2,t25,11(f5.3,5x))
410    format(//,t11,'total',t25,11(f5.3,5x))
       end
       subroutine stress (x,z,hembnk,slope1,width,densty,sigmaz,pi)
c
c
       p = densty * hembnk
c
c           geometry
c
c
       slope = slope1 * pi/180.0
       a = hembnk/tan(slope)
       b = a + width/2.0
       x1 = b - x
c
c           contribution to vertical stress from right half of the load
c
       r2 = sqrt((width/2.0 + a - x1)**2 + z**2)
       r1 = sqrt((x1 - a)**2 + z**2)
       r0 = sqrt(x1**2 + z**2)
       betar = acos((r1**2 + r2**2 - (width/2.0)**2)/(2.0*r1*r2))
       gammar = acos((r1**2 + r0**2 - a**2)/(2.0*r1*r0))
       ciflr = betar + x1/a * gammar - z*(x1-b)/r2**2
c
c
c           contribution to vert. stress from the left side.
c
c
       r1 = sqrt((width + a - x1)**2 + z**2)
       x2 = 2.0 * a + width - x1
       r0 = sqrt(x2**2 + z**2)
       betal = acos((r1**2 +r2**2 - (width/2.0)**2)/(2.0*r1*r2))
       gammal = acos((r1**2 + r0**2 - a**2)/(2.0*r1*r0))
       cifll = betal + x2*gammal/a - z*(x2-b)/r2**2
c
       sigmaz = p/pi*(ciflr + cifll)
       return
       end
```

APPENDIX H

TIME-RATE OF CONSOLIDATION SETTLEMENT

The FORTRAN IV program contained hereafter solves the well known Terzaghi one-dimensional consolidation equation

$$c_v \frac{\delta^2 u}{\delta z^2} = \frac{\delta u}{\delta t} \qquad (H.1)$$

by means of the finite-difference approximation discussed in Chapter IV. Up to 10 contiguous layers with differing values of c_v may be input. Any distribution of initial excess pore pressure may be input because the solution is obtained numerically. The distribution of initial excess pore pressures is input at discrete points. These points are assumed to be evenly spaced within each separate layer. The spacing in each layer is determined with the following equation:

$$\Delta z_i = \frac{\sqrt{c_{vi} \cdot \Delta t}}{\alpha_i} \qquad (H.2)$$

where

$$0 \leq \alpha \leq 1/2$$

i = layer number

c_{vi} = coefficient of consolidation of the i^{th} layer

t = time increment to be used in the analysis

Δz_i = spacing of nodes in the i^{th} layer.

Whenever possible α should be approximately 1/6.

Generally the solution will be more accurate if a large number of nodal points is chosen in each layer.

User Manual - Time Rate of Settlement Program

All input is unformatted.

Units are feet, pounds, and days unless otherwise specified.

1. Read in the number of contiguous consolidating layers (integer)

2. For each layer read in:

   a) the layer number (integer)

   b) c_v of the layer (ft^2/day)

   c) thickness of the layer (ft)

   d) nodal spacing in the layer(ft)

3. Read in the time increment at which excess pore pressures are to be calculated (days)

4. Read in the values of the initial excess pore pressures at each of the nodes from top to bottom (psf)

5. Read in the output scheme: (integer)

   a) If the values of the excress pore pressure at the nodes are desired, input 1. Due to format restrictions this ouput scheme may be used only if 15 or fewer nodes are used.

   b) If the value of the percent consolidation

is desired in each layer, <u>input 2</u>

c) If the value of the percent consolidation is desired for a one layer problem, <u>input 0</u>

6. Read in the:

   a) code for the drainage condition of the top boundary (integer)

   b) code for the drainage condition of the bottom boundary (integer)

   c) depth of the top boundary beneath the ground surface (ft)

   If a boundary is drained, set its drainage condition code = 0

   If a boundary is undrained, set its drainage condition code = 1

7. Read in the termination limits:

   a) the maximum number of years at which the program will cease to calculate the excess pore pressures.

   b) the maximum overall percent consolidation at which the program should cease to calculate excess pore pressures.

The program listing is contained in the following pages.

```
      INTEGER RTOP,RBOTTM,CYCLES,IOUT,NSTRAT(10)
      REAL U(101),UINIT(101),CV(10),THICK(10),DELZ(10),ALPHA(10),
     1     Z11(20),AINTL(10),APRSNL(10),CONSOL(10)
C
C
C          PRINT TITLE
C
      WRITE (6,1)
      WRITE (6,2)
C
C          DIMENSIONS OF UNITS IN INPUT
C          LENGTH -- FEET
C          PRESSURE -- PSF
C          TIME -- DAYS
C                  EXCEPT WHERE SPECIFIED OTHERWISE AS IN THE CASE
C                  OF 'NYEARS' WHICH IS INPUT IN YEARS.
C
C          READ IN THE NUMBER OF SEQUENTIAL COMPRESSIBLE LAYERS
C
      READ (5,*) LAYERS
C
C          FOR EACH LAYER, READ IN:
C          1) THE LAYER NUMBER 'I1'
C          2) THE LAYERS COEFFICENT OF CONSOLIDATION 'CV(I1)',
C          3) THICKNESS 'THICK(I1)'
C          4) THICKNESS INCREMENT 'THICK (I1)'
C
      DO 10 I = 1,LAYERS
                READ (5,*) I1,CV(I1),THICK(I1),DELZ(I1)
10    CONTINUE
C
C          CALCULATE THE NUMBER OF STRATA IN EACH LAYER FOR
C          COMPUTATIONAL PURPOSES.  THE USER SHOULD CHECK THE OUTPUT
C          TO INSURE THAT THE PROGRAM DIVIDES EACH LAYER INTO THE
C          SAME # OF STRATA THAT WAS ASSUMED.
C
      DO 20 I = 1,LAYERS
      VALUE1 = THICK(I)/DELZ(I)
      NSTRAT(I) = INT(VALUE1)
      VALUE2 = FLOAT(NSTRAT(I))
      IF (VALUE2.LT.VALUE1) NSTRAT(I) = NSTRAT(I) + 1
20    CONTINUE
C
C          READ IN THE TIME INCREMENT 'DELT' TO BE USED IN THE ANALYSIS
C
      READ (5,*) DELT
C
C          CALCULATE THE VALUE OF 'ALPHA' FOR EACH LAYER
```

```
C
      DO 30 I = 1,LAYERS
      ALPHA(I) = CV(I)*DELT/DELZ(I)**2
30    CONTINUE
C
C          CALCULATE THE TOTAL # OF STRATA 'NS' IN ALL OF THE LAYERS,
C          AND THE NUMBER OF POINTS 'NPTS' FOR WHICH THE FINITE DIFF-
C          ERENCE PROCEDURE WILL BE PERFORMED.
C
      NS = 0
      DO 40 I = 1,LAYERS
      NS = NS + NSTRAT(I)
40    CONTINUE
      NPTS = NS + 1
C
C          READ IN THE VALUES OF THE INITIAL EXCESS PORE PRESSURE WHICH
C          ARE ASSUMED TO BE EQUAL TO THE TOTAL STRESS CHANGE AT THE
C          POINT FOR EACH OF THE POINTS.  DO NOT REPEAT VALUES AT
C          INTERLAYER BOUNDARIES.
C
C
      READ (5,*) (UINIT(I),I=1,NPTS)
C
C
C          PRINT HEADINGS
C
      WRITE (6,55)
C
C          PRINT INPUT DATA
C
      DO 70 I = 1,LAYERS
            WRITE (6,60) I,THICK(I),NSTRAT(I),DELZ(I),CV(I),ALPHA(I
70    CONTINUE
C
C          CHECK IF  0.0 <  ALPHA  < 0.5
C
      DO 80 I = 1,LAYERS
            IF (ALPHA(I).LE.0.5) GO TO 80
            WRITE (6,75) I
            STOP
80    CONTINUE
C
C          READ IN OUTPUT SCHEME
C          OUTPUT ONLY THE TIME AND THE PERCENT CONSOLIDATION
C          SET  IOUT = 0
C          OUTPUT THE TIME, THE EXCESS PORE PRESSURE AT EACH POINT
C          AND THE PERCENT CONSOLIDATION
C          SET  IOUT = 1
```

```
C           OUTPUT THE TIME ,THE PERCENT CONSOLIDATION IN EACH LAYER
C           AND THE OVERALL PERCENT CONSOLIDATION
C           SET IOUT = 2
C
C           THE USER MUST BE CAREFUL TO NOT USE MORE THAN 15 POINTS IN
C           ORDER TO PREVENT THE OUTPUT FORMATS FROM BLOWING UP WHEN
C           USING IOUT = 1
C
      READ (5,*) IOUT
C
C           INPUT BOUNDARY CONDITIONS OF THE TOP AND BOTTOM OF THE
C                 SOIL LAYER WHOSE TIME RATE OF SETTLEMENT IS BEING
C                 STUDIED.
C
C           IF THE TOP OR BOTTOM BOUNDARY IS DRAINED THEN INPUT
C              RTOP = 0  OR RBOTTM = 0  RESPECTIVELY.
C           IF THE TOP OR BOTTOM BOUNDARY IS UNDRAINED, THEN INPUT
C              RTOP = 1 OR RBOTTM = 1  RESPECTIVELY.
C
      READ (5,*) RTOP,RBOTTM,ZTOP
C
C           INITIALIZATIONS
C
      CNS = 0.0
      T   = 0.0
      AINIT  = 0.0
      APRSNT = 0.0
      IF (LAYERS.EQ.1) GO TO 82
          DO 81 I=1,LAYERS
                  AINTL(I) = 0.
                  APRSNL(I) = 0.
81        CONTINUE
82    CONTINUE
C
C           OUTPUT THE INITIAL PORE PRESSURES AT EACH POINT FROM TOP
C           TO BOTTOM AS IT WAS INPUT.  THIS FORMAT REPEATS THE
C           PRESSURE AT INTER-STRATA BOUNDARIES.
C
      IF (IOUT.GT.0) GO TO 89
      NSTART = 1
      WRITE (6,85)
      DO 95 I = 1,LAYERS
               NEND = NSTRAT(I) + NSTART
               WRITE (6,90) I,(UINIT(J),J=NSTART,NEND)
               NSTART = NEND
95    CONTINUE
      GO TO 101
89    IF (IOUT.GT.1) GO TO 239
      WRITE (6,140)
      Z11(1) = ZTOP
      NP = 1
```

```
      DO 97 I = 1,LAYERS
                NEND = NSTRAT(I)
                DO 96 J = 1,NEND
                NP = NP + 1
                Z11(NP) = Z11(NP-1) + DELZ(I)
96              CONTINUE
97    CONTINUE
      WRITE (6,98) (Z11(NP),NP=1,NPTS)
      WRITE (6,99)
      WRITE (6,150) T,(UINIT(J),J=1,NPTS)
      WRITE (6,165) CNS
      GO TO 101
C
C
C          CALCULATE THE INITIAL AREA OF THE ISOCHRONE,I.E.,THE
C          INTEGRAL OF THE ORDINATES OF EXCESS PORE PRESSURE *
C          THE DEPTH INTERVAL FOR WHICH THAT VALUE OF PRESSURE
C          IS AN AVERAGE VALUE.
C
C
239   WRITE (6,339) (I,I=1,LAYERS)
101   NSTART = 1
      DO 110 I = 1,LAYERS
      NEND = NSTRAT(I) + NSTART
      NEND1 = NEND - 1
      DO 100 J = NSTART,NEND
            IF (J.NE.NSTART) GO TO 102
            AINIT = AINIT + UINIT(J)*DELZ(I)/2.0
            IF (LAYERS.GE.2) AINTL(I) = AINTL(I) + UINIT(J)*DELZ(I)/2.(
            GO TO 100
102         IF (J.GT.NEND1) GO TO 103
            AINIT = AINIT + UINIT(J)*DELZ(I)
            IF (LAYERS.GE.2) AINTL(I) = AINTL(I) + UINIT(J)*DELZ(I)
            GO TO 100
103         AINIT = AINIT + UINIT(J)*DELZ(I)/2.0
            IF (LAYERS.GE.2) AINTL(I) = AINTL(I) + UINIT(J)*DELZ(I)/2.
100   CONTINUE
      NSTART = NEND
110   CONTINUE
C
C
C          CHANGE THE EXCESS PORE PRESSURE AT UNCONFINED BOUNDARIES
C          TO THE AMBIENT VALUE = 0.5 * INITIAL EXCESS PRESSURE.
C
C
      IF (RTOP.EQ.0.AND.RBOTTM.EQ.1) GO TO 111
      IF (RTOP.EQ.1.AND.RBOTTM.EQ.0) GO TO 112
      IF (RTOP.EQ.0.AND.RBOTTM.EQ.0) GO TO 113
      IF (RTOP.EQ.1.AND.RBOTTM.EQ.1) GO TO 114
111   UINIT(1) = UINIT(1)/2.0
      APRSNT = AINIT - UINIT(1)*DELZ(1)/2.0
```

```
      IF (LAYERS.GE.2) APRSNL(1) = AINTL(1) - UINIT(1)*DELZ(1)/2.0
      GO TO 120
112   UINIT(NPTS) = UINIT(NPTS)/2.0
      APRSNT = AINIT - UINIT(NPTS)*DELZ(LAYERS)/2.0
      IF (LAYERS.GE.2) APRSNL(LAYERS) = APRSNL(LAYERS) - UINIT(NPTS)
     1                                  *DELZ(LAYERS)/2.0
      GO TO 120
113   UINIT(1) = UINIT(1)/2.0
      UINIT(NPTS) = UINIT(NPTS)/2.0
      APRSNT = AINIT - (UINIT(1)*DELZ(1) + UINIT(NPTS)*DELZ(LAYERS))/2.(
      IF (LAYERS.GE.2) APRSNL(1) = AINTL(1) - UINIT(1)*DELZ(1)/2.0
      IF (LAYERS.GE.2) APRSNL(LAYERS) = APRSNL(LAYERS) - UINIT(NPTS)
      GO TO 120
116   WRITE (6,117)
      STOP
120   CNS = (1.0 - APRSNT/AINIT)*100.0
      IF (LAYERS.EQ.1) GO TO 126
      DO 125 I = 1,LAYERS
           CONSOL(I) = (1.0 - APRSNL(I)/AINTL(I))*100.
125   CONTINUE
126   CONTINUE
C
      IF (IOUT.EQ.0) WRITE (6,130)
      IF (IOUT.NE.0) GO TO 114
      WRITE (6,145) T,CNS
      GO TO 115
114   IF (IOUT.NE.1) GO TO 115
      WRITE (6,150) T,(UINIT(J),J=1,NPTS)
      WRITE (6,165) CNS
C
C          INPUT THE ITERATION LIMITS
C          NYEARS = # OF YEARS THE PROGRAM WILL CALCULATE EXCESS PORE
C                   PRESSURES AND CONSOLIDATIONS.
C          CMAX   = THE MAXIMUM PERCENT OF CONSOLIDATION THE PROGRAM
C                   WILL CALCULATE PRESSURES AND CONSOLIDATIONS.
C
115   READ(5,*) NYEARS,CMAX
C
      CYCLES = NYEARS*365/INT(DELT)
C
C
C
C                             FINITE DIFFERENCE LOOP
C
C
C        CALCULATE EXCESS PORE PRESSURES AND PERCENT CONSOLIDATION
C        FOR EACH TIME STEP
C
```

```
C
      DO 200 ICYCLE = 1,CYCLES
      T = T + DELT
      NSTART = 1
C
C          CALCULATE THE EXCESS PORE PRESSURE AT ALL THE POINTS
C          FOR THE ICYCLE'TH CYCLE
C
      DO 160 I = 1,LAYERS
      NEND = NSTRAT(I) + NSTART
C
C          INTERNAL POINTS IN EACH LAYER
C
      DO 155 J = NSTART,NEND
                IF ((I.EQ.1.AND.J.EQ.1)) GO TO 155
                IF (J.LT.NEND) U(J) = ALPHA(I)*(UINIT(J+1)+UINIT(J-1))
     1                                 + (1.0 - 2.0*ALPHA(I))*UINIT(J)
155   CONTINUE
      NSTART = NEND
160   CONTINUE
C
C          INTER LAYER BOUNDARY POINTS
C          IMPOSE CONTINUITY OF FLOW ACROSS THE BOUNDARY
C
      NFACE = 1
      NL1 = LAYERS - 1
      IF (NL1.LE.0) GO TO 181
      DO 180 I = 1,NL1
                NFACE = NFACE + NSTRAT(I)
                U(NFACE) = U(NFACE + 1) - (U(NFACE + 1) - U(NFACE - 1)
     1                       /(1.0 + (CV(I+1)/CV(I))*(DELZ(I)/DELZ(I+1))
180   CONTINUE
C
C          CALCULATE THE PORE PRESSURE AT THE TOP AND BOTTOM OF THE
C          SEQUENCE OF COMPRESSIBLE LAYERS. IT IS ASSUMED THAT
C          UNDRAINED BOUNDARIES MAY BE SIMULATED WITH A MIRROR
C          IMAGE OF THE EXISTING PORE PRESSURES ON THE OTHER SIDE
C          OF THE BOUNDARY.
C
C
181   IF (RTOP.EQ.0) U(1)=0.0
      IF (RTOP.EQ.1) U(1) = 2.0*ALPHA(1)*UINIT(2) +
     1                      (1.0 - 2.0*ALPHA(1))*UINIT(1)
C
      IF (RBOTTM.EQ.0) U(NPTS) = 0.0
      IF (RBOTTM.EQ.1)  U(NPTS) = 2.0*ALPHA(LAYERS)*UINIT(NPTS-1) +
     1                            (1.0 - 2.0*ALPHA(LAYERS))*UINIT(NPTS)
```

```
C
C          CALCULATE THE PERCENT CONSOLIDATION
C
      APRSNT = 0.0
      IF (LAYERS.EQ.1) GO TO 186
      DO 185 I = 1,LAYERS
                 APRSNL(I) = 0.
185   CONTINUE
186   CONTINUE
      NSTART = 1
C
C          CALCULATE THE PRESENT ISOCHRONE
C
      DO 195 I = 1,LAYERS
      NEND = NSTRAT(I) + NSTART
      NEND1 = NEND - 1
C
      DO 190 J = NSTART,NEND
                 IF (J.NE.NSTART) GO TO 1181
                 APRSNT = APRSNT + U(J)*DELZ(I)/2.0
                 IF (LAYERS.EQ.2) APRSNL(I) = APRSNL(I) + U(J)*
     1                                        DELZ(I)/2.0
                 GO TO 190
1181             IF (J.GT.NEND1) GO TO 182
                 APRSNT = APRSNT + U(J)*DELZ(I)
                 IF (LAYERS.EQ.2) APRSNL(I) = APRSNL(I) + U(J)*
     1                                        DELZ(I)
                 GO TO 190
182              IF (J.EQ.NEND) APRSNT = APRSNT + U(J)*DELZ(I)/2.0
                 IF (J.EQ.NEND.AND.LAYERS.EQ.2)
     1               APRSNL(I)= APRSNL(I) + U(J)*DELZ(I)/2.0
190   CONTINUE
      NSTART = NEND
195   CONTINUE
      CNS = (1.0 - APRSNT/AINIT)*100.0
      IF (LAYERS.EQ.1) GO TO 1196
      DO 1195 I = 1,LAYERS
                 CONSOL(I) = (1.0 -APRSNL(I)/AINTL(I))*100.
1195  CONTINUE
1196  CONTINUE
C
C          OUTPUT THE TIME,THE PERCENT CONSOLIDATION,
C          AND THE EXCESS PORE PRESSURE AT EACH NODE
C
      IF (IOUT.GT.0) GO TO 196
      WRITE (6,145) T,CNS
      GO TO 197
196   IF (IOUT.GT.1) GO TO 1197
      WRITE (6,150) T,(U(J),J=1,NPTS)
      WRITE (6,165) CNS
      GO TO 197
1197  WRITE (6,1200) T,(CONSOL(I),I=1,LAYERS)
```

```
C
C          CHECK TERMINATION LIMITS
C
197   IF (CNS.GT.CMAX) GO TO 300
      IF (ICYCLE.EQ.CYCLES) GO TO 300
C
C          RESET THE EXCESS PORE PRESSURES AT ALL THE POINTS
C          FOR THE NEXT TIME CYCLE
C
      DO 199 I = 1,NPTS
      UINIT(I) = U(I)
199   CONTINUE
200   CONTINUE
C
300   CONTINUE
C
C          FORMATS
C
1     FORMAT (1H1,30(/),T50,'TIME RATE OF SETTLEMENT PROGRAM',/,
     1        T52,'PROGRAMMED BY MARTY GOODMAN',///)
2     FORMAT (1H1,20(/))
55    FORMAT (T22,'LAYER ',T31,'THICKNESS',T43,'# OF STRATA',T58,
     1        'STRATA THICKNESS',T78,'COEFF. OF CONSOLIDATION',T105,
     2        'ALPHA',///)
60    FORMAT (T23,I2,T33,F5.1,T47,I2,T64,F5.2,T85,F8.5,T104,F6.4)
75           FORMAT (' LAYER ',I2,' HAS A VALUE OF ALPHA MORE THAN'
     1          ' 0.5',/,' EITHER REDUCE THE TIME INCREMENT DELT',/,T20
     2          'OR',/,
     3          ' REDUCE THE # OF STRATA IN THE LAYERS WITH CALCULATED'
     4          ' VALUES OF ALPHA GREATER THAN 0.5',/,
     5          ' THIS MAY BE ACCOMPLISHED BY INCREAING DELZ')
85    FORMAT (1H1,T5,'LAYER',T20,'INITIAL EXCESS PORE PRESSURES',//)
90    FORMAT (5X,I2,5X,18(F5.0,1X))
98    FORMAT (10X,18(1X,F6.0))
99    FORMAT (2(/))
117   FORMAT (' THE TOP OF AND BOTTOM BOUNDARIES ARE UNDRAINED',/,
     1        ' NO COSOLIDATION WILL OCCUER')
130   FORMAT (1H1,5(/),T43,'TIME (DAYS)',T67,'PERCENT CONSOLIDATION'///
140   FORMAT(1H1,5(/),T2,'TIME (DAYS)',T50,'EXCESS PORE PRESSURE (PSF)'
     1       T120,'CONSOLIDATION',/,T59,'Z (FEET)',/)
145   FORMAT (T40,F10.1,20X,F10.2)
150   FORMAT (F8.1,2X,15(1X,F6.0))
165   FORMAT (1H+,T119,F10.2)
339   FORMAT(1H1,///,T11,'TIME',T50,'LAYERS',//,T25,10(I4,6X),//)
1200  FORMAT(T5,F10.1,8X,10(F8.1,2X))
      END
```

COVER DESIGN BY ALDO GIORGINI